

Assignment

Class-12

Subject: Mathematics

Unit 2

Complex Numbers

Part - A

I. One mark questions

Fill in the blanks:

- $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is _____
- The conjugate of a complex number is $\frac{1}{i-2}$ then, the complex number is _____
- If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to _____
- If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is _____
- If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is _____

Choose the best answers:

- If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is
(i) 1 (ii) 2 (iii) 3 (iv) 4
- z_1, z_2 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is
(i) 3 (ii) 2 (iii) 1 (iv) 0
- If $z = x + iy$ is a complex number such that $|z + 2| = |z - 2|$, then the locus of z is
(i) real axis (ii) imaginary axis (iii) ellipse (iv) circle
- The principle of argument of $\frac{3}{-1+i}$ is
(i) $\frac{-5\pi}{6}$ (ii) $\frac{-2\pi}{3}$ (iii) $\frac{-3\pi}{4}$ (iv) $\frac{-\pi}{2}$
- If α and β are the roots of $x^2 + x + 1 = 0$ then $\alpha^{2020} + \beta^{2020}$ is
(i) -2 (ii) -1 (iii) 1 (iv) 2

Part - B

II. Very Short Answer.

- Simplify the following $i^{59} + \frac{1}{i^{59}}$
- Write $\frac{3+4i}{5-12i}$ in the $x + iy$ form, hence find its real and imaginary parts
- If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$
- Find the square root of $6 - 8i$
- Simplify $\left(\sin \frac{\pi}{6} + i \cos \frac{\pi}{6}\right)^{18}$

Part – C

III. Short Answer.

1. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form
2. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions
3. If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus number of z is real axis
4. Write in polar form: $-2 - i2$
5. If $\omega \neq 1$ is a cube root of unity, show that $(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$

Part – D

IV. Write in detail.

1. If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$
2. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

Unit – 3 - Theory of Equations

Part - A

I. One mark questions

1. A zero of $x^2 + 64$ is _____
2. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f \circ g)(x)$, then the degree of h is _____
3. A polynomial equation in x of degree n always has _____
4. If α, β and γ are the zeroes of $x^2 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is _____
5. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$ _____
6. The polynomial $x^3 - kx^2 + 9x$ has three real zero if and only if, k satisfies
(i) $|k| \leq 6$ (ii) $k = 0$ (iii) $|k| > 6$ (iv) $|k| \geq 6$
7. The number of real number in $(0, 2\pi)$ satisfying $\sin^4 x - 2 \sin^2 x + 1$ is
(i) 2 (ii) 4 (iii) 1 (iv) ∞

8. If $x^2 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if
- (i) $a \geq 0$ (ii) $a > 0$ (iii) $a < 0$ (iv) $a \leq 0$
9. The polynomial $x^3 + 2x + 3$ has
- (i) one negative and two imaginary zeroes
(ii) one positive and two imaginary zeroes
(iii) three real zeroes
(iv) no zeroes
10. The number of positive zeroes of the polynomial $\sum_{j=0}^n {}^n C_j (-1)^j x^j$ is
- (i) 0 (ii) n (iii) $< n$ (iv) r

Part - B

II. Very short answer

- Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root
- If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.
- Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.
- Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root.
- Show the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x .

Part - C

III. Short answer

- If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{1}} = 0$$

- Find the roots of $2x^2 + 3x^2 + 2x + 3 = 0$
- Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

4. If α, β and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

5. Solve the cubic equations: (i) $2x^3 - 9x^2 + 10x - 3 = 0$ (ii) $8x^3 - 2x^2 - 7x + 3 = 0$

Part - D

IV. Answer in Detail

1. If $2 + i$ and $3, \sqrt{2}$ are roots of the equation

$x^6 - 1x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots

2. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.