29.07.21

Determinant :-

For Every Square Matrix 'A', We can associate a real no is called as Determinant and it is denoted by det(A) (or) $|A|$ or Δ.

↳ Read as determinant of A.

✓ Matrix is a structure alone but determinant is a value of Matrix.

✓ Det. can exist for square Matrix.

Methods in Determinant

- Expansion Method
- (By pty method) (or) (w/o expansion method)

For 2x2 Determinant :-

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$|A| = P.M.D - P.S.D$

$$|A| = ad - bc$$

eg:-

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 5 \end{bmatrix}$$

Sol:-

$$|A| = \begin{vmatrix} 1 & 4 \\ 3 & 5 \end{vmatrix}$$

$= 5 - 12$

~~|A| = -2 ≠ 0~~

$|A| = -7 \neq 0$

For 3x3 :-

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}_{3\times 3}$$

Sol:-

$$|A| = \begin{vmatrix} \overset{+}{a} & \overset{-}{b} & \overset{+}{c} \\ d & e & f \\ g & h & i \end{vmatrix}$$

(Expanding any rows or columns)

Expanding 'R_1'

$$= a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= a(ei - hf) - b(di - gf) + c(dh - ge)$$

$$|A| = a(ei - hf) - b(di - gf) + c(dh - ge)$$

Minor :- (M_{ij})

Let the Square Matrix ~~of~~ be A.
Then By Deleting corresponding row and Column of an element a_{ij} is called as Minor of A & it is denoted by M_{ij}.

Let $A = \begin{bmatrix} 1 & 2 \\ -4 & -5 \end{bmatrix}$

Sol:

$\boxed{|\ | - det}$

Minor of 1 is $|-5| = -5$

Minor of 2 is $|-4| = -4$

Minor of -4 is $|2| = 2$

Minor of -5 is $|1| = 1$.

$$M = \begin{bmatrix} -5 & -4 \\ 2 & 1 \end{bmatrix}$$

It is Called as Minor of A.

Co-factor :- (A_{ij})

Let A be square Matrix of order n.
Then Co-factor of A_{ij} is defined as
Product of Signed & Minor M_{ij}

$$A_{ij} = (-1)^{i+j} M_{ij}$$

eg:

$$\begin{array}{cc} + & - \\ \end{array}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{array}{cc} - & + \end{array}$$

$a_{12} = (-1)^{i+j}$
$= (-1)^{1+2}$
$= (-1)^3$
$= -$

$a_{21} = (-1)^{2+1}$
$= (-1)^3$
$a_{21} = -1$

$a_{22} = (-1)^{2+2}$
$= (-1)^4$
$a_{22} = 1$

$A_{ij} = A_{11} = (-1)^{1+1} \underline{M_{11}}$
$= (-1)^2 a_{22}$

$$\boxed{A_{11} = + a_{22}}$$

* For 2x2

$$\begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

* For 3x3

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

For 2×2

$$A = \begin{bmatrix} 3 & -4 \\ -5 & 6 \end{bmatrix} \qquad \begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

Co-factor of $3 = +|6| = 6$

Co-factor of $-4 = -(-5) = 5$

Co-factor of $-5 = -(-4) = 4$

Co-factor of $6 = +|3| = 3$.

Co-factor of $A = A_{ij} = \begin{bmatrix} 6 & 5 \\ 4 & 3 \end{bmatrix}$

For 3×3

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Sol.

Co-factor of $1 = + \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = +(2-0)$

$= 2$.

Co-factor of $2 = - \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} = -(0-0) = -0$

$= 0$.

For 3×3 Determinant:-

It is named as Rangoli Method.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$|A| = \begin{array}{ccccc} & & h & & \\ & a & b & c & \\ f & d & e & f & d \\ & g & h & i & \\ & & b & & \end{array}$$

P.M.D − P.S.D.

$$|A| = (fgb + aei + hcd) - (fah + gec + bid)$$

For 3×3 Determinant:-

Expansion of any row/column Method

$$A = \begin{bmatrix} \overset{+}{a} & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$|A| = \begin{vmatrix} \overset{+}{a} & \overset{-}{b} & \overset{+}{c} \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$|A| = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Property of Determinant:-

Pty 1:-

In a det, Interchanging a row into Column or vice-versa, the value of det is Unaltered. (unchanged).

eg:

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \qquad \Delta_1 = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

$\Delta = ad - bc$

$\Delta_1 = ad - bc$

$\Rightarrow \Delta = \Delta_1$ (or) $|A| = |A^T|$

Pty 2:-

In a det, Interchanging a row into row or Column into column, the value of det is unchanged but sign will be Change.

eg:-

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \qquad \Delta_1 = \begin{vmatrix} c & d \\ a & b \end{vmatrix} \; R_1 \leftrightarrow R_2$$

$\Delta = ad - bc$

$\Delta_1 = cb - ad$

$= -ad + bc$

$\Delta_1 = -(ad - bc)$

$\Delta_1 = -\Delta$.

Pty 3:-

In a det, any two rows or columns are identical (same), then value of det is zero.

eg:-

$$\Delta = \begin{vmatrix} a & b \\ a & b \end{vmatrix} \quad R_1 \equiv R_2$$

$$\Delta = ab - ab$$

$$\Delta = 0.$$

Pty 4:-

In a det, Multiply a scalar in any rows or columns, the value of det is scalar times of that det value.

eg:

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\Delta = ad - bc$$

$$\Delta_1 = K \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} a & b \\ Kc & Kd \end{vmatrix}$$

$$= aKd - Kcb$$

$$\Delta_1 = K(ad - bc)$$

$$\Delta_1 = K\Delta.$$

Pty 5:-

In a det, any rows or columns has sum or diff of two or more elts, then the det can be expressed as sum or diff of two or more det.

eg:-

$$\Delta = \begin{vmatrix} a+b & c+d \\ e & f \end{vmatrix}$$

$$\Delta = (a+b)f - e(c+d)$$

$$\Delta = af + bf - ec - ed.$$

$$\Delta = \begin{vmatrix} a & c \\ e & f \end{vmatrix} + \begin{vmatrix} b & d \\ e & f \end{vmatrix}$$

$$= af - ec + bf - ed$$

Pty 6:-

In a det, one row is added/sub ~~Multiplied~~ to another row/column which is Multiply by scalar or not a scalar.

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\Delta_1 = \begin{vmatrix} a+c & b+d \\ c & d \end{vmatrix} \quad R_1 \to \boxed{R_1} + R_2$$

$$= (a+c)d - (b+d)c$$

$$= ad + \cancel{cd} - bc - \cancel{dc}$$

$$\Delta_1 = ad - bc.$$

Ex: 7.2.

1) Without expanding the determinant,

P.T $\begin{vmatrix} s & a^2 & b^2+c^2 \\ s & b^2 & c^2+a^2 \\ s & c^2 & a^2+b^2 \end{vmatrix} = 0$

Sol:-

$$\Delta = \begin{vmatrix} s & a^2 & b^2+c^2 \\ s & b^2 & c^2+a^2 \\ s & c^2 & a^2+b^2 \end{vmatrix}$$

Take 's' as common in C_1

$$= s \begin{vmatrix} 1 & a^2 & a^2+b^2+c^2 \\ 1 & b^2 & a^2+b^2+c^2 \\ 1 & c^2 & a^2+b^2+c^2 \end{vmatrix} \quad C_3 \to C_3 + C_2$$

$$= s(a^2+b^2+c^2) \begin{vmatrix} 1 & a^2 & 1 \\ 1 & b^2 & 1 \\ 1 & c^2 & 1 \end{vmatrix} \quad C_1 \equiv C_3$$

$$= s(a^2+b^2+c^2)(0)$$

$$= 0.$$

5.8

2) S.T $\begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix} = 0.$

$(x + yx)$

$x(1+y)$

Sol:-

$$\Delta = \begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}$$

(×) and (÷) abc

$$= \frac{abc}{abc} \begin{vmatrix} b+c & bc & b^2c^2 \\ c+a & ca & c^2a^2 \\ a+b & ab & a^2b^2 \end{vmatrix}$$

(×) a in R_1, (×) b in R_2, (×) c in R_3

$$= \frac{1}{abc} \begin{vmatrix} a(b+c) & abc & ab^2c^2 \\ b(c+a) & abc & a^2bc^2 \\ c(a+b) & abc & a^2b^2c \end{vmatrix}$$

Take abc from C_2 & abc from C_3

$$= \frac{1}{\cancel{abc}} (\cancel{abc})(abc) \begin{vmatrix} ab+ca & 1 & bc \\ bc+ab & 1 & ac \\ ca+bc & 1 & ab \end{vmatrix}$$

$$= abc \begin{vmatrix} ab+bc+ca & 1 & bc \\ ab+bc+ca & 1 & ac \\ ab+bc+ca & 1 & ab \end{vmatrix} \quad C_1 \to C_1 + C_3$$

Take $(ab+bc+ca)$ from C_1

$$= abc(a+b+c) \begin{vmatrix} 1 & 1 & bc \\ 1 & 1 & ac \\ 1 & 1 & ab \end{vmatrix} \quad C_1 \equiv C_2$$

$$= (abc)(a+b+c)(0)$$

$$= 0.$$

5): P.T $\begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix} = 0.$

Sol:-

$\boxed{\sec^2\theta - \tan^2\theta = 1}$

$\otimes -1$

$-\sec^2\theta + \tan^2\theta = -1$

$$\Delta = \begin{vmatrix} \sec^2\theta & \tan^2\theta & 1 \\ \tan^2\theta & \sec^2\theta & -1 \\ 38 & 36 & 2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \sec^2\theta - \tan^2\theta & \tan^2\theta & 1 \\ \tan^2\theta - \sec^2\theta & \sec^2\theta & -1 \\ 38-36 & 36 & 2 \end{vmatrix} \quad C_1 \to C_1 - C_2$$

$$= \begin{vmatrix} 1 & \tan^2\theta & 1 \\ -1 & \sec^2\theta & -1 \\ 2 & 36 & 2 \end{vmatrix} \quad C_1 \equiv C_3$$

$$\Delta = 0 \quad \#$$

6). S.T $\begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix} = 0$

Sol:

$$\Delta = \begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix}$$

$$\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ a & b & c \end{vmatrix} \quad \begin{array}{l} R_1 \to R_1 - R_2 - R_3 \\ R_1 \equiv R_3 \end{array}$$

$\cancel{x}+2a$
$-\cancel{x}$
$-a$
a

$$\Delta = 0.$$

Solution:-

$$\Delta = \begin{vmatrix} x+2a & y+2b & z+2c \\ x & y & z \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ x & y & z \\ a & b & c \end{vmatrix} + \begin{vmatrix} 2a & 2b & 2c \\ x & y & z \\ a & b & c \end{vmatrix}$$

$R_1 \equiv R_2$

$$= 0 + 2 \begin{vmatrix} a & b & c \\ x & y & z \\ a & b & c \end{vmatrix} \quad R_1 \equiv R_3$$

$$= 0 + 2(0)$$

$$\Delta = 0 \ \#$$

5M.

3) P.T $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

Sol:-

$$\Delta = \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & bc & c(a+c) \\ a(a+b) & b^2 & ac \\ ab & b(b+c) & c^2 \end{vmatrix}$$

Take a from C_1, b from C_2 & c from C_3

$$= abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$$= abc \begin{vmatrix} -b & c-b & c \\ a+b & b & a \\ b & b+c & c \end{vmatrix} \quad R_1 \to R_1 - R_2$$

$$= abc \begin{vmatrix} 0 & 2c & 2c \\ a & -c & a-c \\ b & b+c & c \end{vmatrix} \begin{array}{l} R_1 \to R_1 + R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$2c - 2c = 0$

$\cancel{c} - a + \cancel{c} = a$

$b + \cancel{c} - \cancel{c} = b$

$$= abc \begin{vmatrix} \overset{+}{0} & \overset{-}{0} & \overset{+}{2c} \\ a & -a & a-c \\ b & b & c \end{vmatrix} \quad C_2 \to C_2 - C_3$$

Expand R_1

P.M.D.

P.S.D

$$= abc \left[0 - 0 + 2c \begin{vmatrix} a & -a \\ b & b \end{vmatrix} \right]$$

$$= abc \left[2c(ab - (-ab)) \right]$$

$$= abc \left[2c(ab + ab) \right]$$

$$= abc \left[2c(2ab) \right]$$

$$= 4a^2b^2c^2$$

H.P.

10-08-2021

Say

4) P.T

α

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$$

Sol:-

$$\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

R_1 $\cancel{1}+a$ $\cancel{1}$ 1

R_2 $\cancel{1}$ $\cancel{1}+b$ 1

a $-b$ 0

$$\Delta = \begin{vmatrix} a & -b & 0 \\ 0 & b & -c \\ 1 & 1 & 1+c \end{vmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

R_2 1 $1+b$ 1

R_3 1 1 $1+c$

0 b $-c$

Expand 'R_1'

P.M.D

P.S.D)

$$\Delta = a\begin{vmatrix} b & -c \\ 1 & 1+c \end{vmatrix} + b\begin{vmatrix} 0 & -c \\ 1 & 1+c \end{vmatrix} + 0.$$

$$= a[b(1+c)+c] + b(0+c) + 0.$$

$$= a(b+bc+c) + b(c)$$

$$= ab + abc + ca + bc.$$

$$= abc\left(\frac{1}{c} + 1 + \frac{1}{b} + \frac{1}{a}\right)$$

$$= abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$ H

9) P.T $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

Sol :

$$\Delta = \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$$

ply 5

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

(×) and (÷) abc

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{abc}{abc}\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

(×) a in R_1
b in R_2
c in R_3

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{1}{abc}\begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$

Take abc common from C_3

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \frac{\cancel{abc}}{\cancel{abc}}\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix}$$

$C_1 \leftrightarrow C_3$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (-) \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix}$$

$C_2 \leftrightarrow C_3$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + (-) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$= 0$ #

Aliter:

P.T $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$

Sol W.K.T $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

& $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$

$$\Delta = \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$= (a-b)(b-c)(c-a) - (a-b)(b-c)(c-a)$$

$$\Delta = 0.$$

11). S.T $\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by x^4.

Sol:

$\cancel{(3+4+6)}$

$\cancel{(3+9+6)}$

$3\cancel{(1+3+2)}$

$$\Delta = \begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$$

$\times$ and $\div$ abc

$$= \frac{abc}{abc}\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$$

$\times$ a in R_1, b in R_2, c in R_3

$$= \frac{1}{abc}\begin{vmatrix} a(a^2+x^2) & a^2b & a^2c \\ ab^2 & b(b^2+x^2) & b^2c \\ ac^2 & bc^2 & c(c^2+x^2) \end{vmatrix}$$

Take a from C_1, b from C_2, C from C_3

$$= \frac{abc}{abc}\begin{vmatrix} a^2+x^2 & a^2 & a^2 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+c^2+x^2 & a^2+b^2+c^2+x^2 & a^2+b^2+c^2+x^2 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix} \quad R_1 \to R_1+R_2+R_3$$

Take $a^2+b^2+c^2+x^2$ common from R_1

$$= (a^2+b^2+c^2+x^2)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+x^2 & b^2 \\ c^2 & c^2 & c^2+x^2 \end{vmatrix}$$

$$= (a^2+b^2+c^2+x^2)\begin{vmatrix} 0 & 0 & 1 \\ -x^2 & x^2 & b^2 \\ 0 & -x^2 & c^2+x^2 \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

Expand R_1

$$= (a^2+b^2+c^2+x^2)\left[0 - 0 + 1\begin{vmatrix} -x^2 & x^2 \\ 0 & -x^2 \end{vmatrix}\right]$$

$$= (a^2+b^2+c^2+x^2)\left[1(x^4 - 0)\right]$$

$$= (a^2+b^2+c^2+x^2)x^4$$

It is divisible by x^4

12-08-2021

7) Write the general form of 3x3 Skew Symmetric Matrix & prove it's determinant is zero.

Sol:

$$A = \begin{bmatrix} 0 & a_{21} & a_{22} \\ -a_{21} & 0 & a_{23} \\ -a_{22} & -a_{23} & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$ is skew-symmetric Matrix

$$\Delta = |A| = \begin{vmatrix} \overset{+}{0} & \overset{-}{a} & \overset{+}{b} \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

Expand R_1

$$= 0 - a\begin{vmatrix} -a & c \\ -b & 0 \end{vmatrix} + b\begin{vmatrix} -a & 0 \\ -b & -c \end{vmatrix}$$

P.M.D

P.S.D

$$= 0 - a(0 + bc) + b(ac - 0)$$

$$= -a(bc) + b(ac)$$

$$= -abc + abc.$$

$$\Delta = |A| = 0.$$

8). If $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$

P.T a, b, c are in G.P or

α is a root of $ax^2+2bx+c=0$.

$$\Rightarrow \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0.$$

$$\Rightarrow \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha & b\alpha & -b\alpha-c \end{vmatrix} = 0 \quad R_3 \Rightarrow R_3 - R_2$$

$$\begin{array}{lllll} R_3 & a\alpha & \cancel{+b} & b\alpha\cancel{+c} & 0 \\ R_2 & (-)\cancel{b} & & (-)\cancel{c} & (-)b\alpha+c \\ \hline & a\alpha & & b\alpha & -b\alpha-c \end{array}$$

$$\Rightarrow \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & -(a\alpha^2+2b\alpha+c) \end{vmatrix} = 0 \quad R_3 \to R_3 - \alpha R_1$$

$+ \quad - \quad +$

$$\begin{array}{llll} R_3 & \cancel{a\alpha} & \cancel{b\alpha} & -b\alpha-c \\ \alpha R_1 & (-)\cancel{a\alpha} & (-)\cancel{b\alpha} & (-)\, 2(-) \ a\alpha^2+b\alpha \\ \hline & 0 & 0 & -2b\alpha \\ & & & -a\alpha^2-c \end{array}$$

$= 0 \quad 0 \quad -(a\alpha^2+2b\alpha+c)$

Expand R_3

$$\Rightarrow 0 - 0 - (a\alpha^2+2b\alpha+c)\begin{vmatrix} a & b \\ b & c \end{vmatrix} = 0$$

$$-(a\alpha^2+2b\alpha+c)(ac-b^2)=0.$$

$\Rightarrow (a\alpha^2+2b\alpha+c)=0$; $ac-b^2=0$

$\underline{a\alpha^2+2b\alpha+c=0}$; $b^2=ac$

α is root of

$ax^2+2bx+c=0$

If a, b, c are in G.p then $b^2=ac$.

Hence proved.

$\frac{b}{a} = \frac{c}{b}$

$\boxed{b^2=ac}$

$\underline{a, b, c \text{ are in G.p}}$

10). If a, b, c are p^{th}, q^{th} & r^{th} term of a A.p. find the value of

$$\begin{vmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

Sol:

$t_n = a+(n-1)d$

$t_n = A+(n-1)d \rightarrow (1)$

Gn: $t_p = a$; $t_q = b$; $t_r = c$

$n=p$ in (1)	$n=q$ in (1)	$n=r$ in (1)
$t_p = A+(p-1)d$	$t_q = A+(q-1)d$	$t_r = A+(r-1)d$
$\underline{a = A+(p-1)d.}$	$b = A+(q-1)d$	$c = A+(r-1)d$

$$\begin{vmatrix} a & b & c \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} A+(p-1)d & A+(q-1)d & A+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} A & A & A \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} (p-1)d & (q-1)d & (r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

Take A from R_1 & d from R_1 in 2^{nd} det.

$$= A \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + d \begin{vmatrix} p-1 & q-1 & r-1 \\ p-1 & q-1 & r-1 \\ 1 & 1 & 1 \end{vmatrix} \begin{matrix} \\ R_2 \to R_2 - R_3 \\ \\ \end{matrix}$$

$R_1 \equiv R_3$ $\qquad$ $R_1 \equiv R_2$

$= A(0) + d(0)$

$= 0$ #

17.08.21

12) If a, b, c are all +ve & p^{th}, q^{th}, r^{th} terms of a G.P. S.T $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$

Sol:

$\boxed{t_n = ar^{n-1}}$

G.T of G.p is $t_n = AR^{n-1}$

Gn:- $t_p = a$, $n = p$, $t_p = AR^{p-1}$, $a = AR^{p-1}$

Take log on b.s

$\log a = \log(AR^{p-1})$

$\log a = \log A + \log R^{p-1}$

$\log a = \log A + (p-1)\log R$

$t_q = b$, $n = q$, $t_q = AR^{q-1}$, $b = AR^{q-1}$

Take log on b.s

$\log b = \log(AR^{q-1})$

$\log b = \log A + \log R^{q-1}$

$\log b = \log A + (q-1)\log R$

$t_r = c$, $n = r$, $t_r = AR^{r-1}$, $c = AR^{r-1}$

Take log on b.s

$\log c = \log(AR^{r-1})$

$\log c = \log A + \log R^{r-1}$

$\log c = \log A + (r-1)\log R.$

$\Rightarrow \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix}$ ~~$= \log A +$~~

$= \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$

$$= \begin{vmatrix} \log A & p & 1 \\ \log A & q & 1 \\ \log A & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$$

Take log A from C_1 Take log R from C_1 in II

$C_2 \to C_2 - C_3$

$$= \log A \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$$

$C_1 \equiv C_3$ $C_1 \equiv C_2$

$$= \log A(0) + \log R(0)$$

$$= 0 + 0$$

$$= 0$$ #

13). Find the value of

$$\begin{array}{ccc} + & - & + \\ \end{array}$$
$$\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix} \quad \text{if } x, y, z \neq 1.$$

Sol:

Expanding any rows (R_1)

$$= 1\begin{vmatrix} 1 & \log_y z \\ \log_z y & 1 \end{vmatrix} - \log_x y \begin{vmatrix} \log_y x & \log_y z \\ \log_z x & 1 \end{vmatrix}$$
$$+ \log_x z \begin{vmatrix} \log_y x & 1 \\ \log_z x & \log_z y \end{vmatrix}$$

$$= 1\left(1 - \log_z y \log_y z\right) - \log_x y\left(\log_y x - \log_z x \log_y z\right)$$
$$+ \log_x z\left(\log_y x \log_z y - \log_z x\right).$$

$$= 1\left(1 - \log_z z\right) - \log_x y\left(\cancel{\log_y x} - \cancel{\log_y x}\right)$$
$$+ \log_x z\left(\log_z x - \log_z x\right)$$

$$= 1(1-1) - \log_x y\,(0) + \log_x z\,(0)$$

$$= 0 \;/\!/.$$

19.08.21

14). If $A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$; ~~P.T~~

P.T $\sum_{k=1}^{n} \det(A^k) = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$.

Sol:- T.P:- $\sum_{k=1}^{n} |A^k| = \frac{1}{3}\left(1-\frac{1}{4^n}\right)$ $\quad |A|^n = |A^n|$

$\Rightarrow |A^1| + |A^2| + |A^3| + \dots + |A^n| = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$

$\Rightarrow |A|^1 + |A|^2 + |A|^3 + \dots + |A|^n = \frac{1}{3}\left(1 - \frac{1}{4^n}\right)$

$A = \begin{bmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{bmatrix}$

$|A| = \begin{vmatrix} \frac{1}{2} & \alpha \\ 0 & \frac{1}{2} \end{vmatrix} = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) - 0 = \frac{1}{4}$

L.H.S: $\Rightarrow |A|^1 + |A|^2 + |A|^3 + \dots + |A|^n$

$\Rightarrow \underset{t_1}{\frac{1}{4}} + \underset{t_2}{\left(\frac{1}{4}\right)^2} + \underset{t_3}{\left(\frac{1}{4}\right)^3} + \dots + \underset{t_n}{\left(\frac{1}{4}\right)^n}$.

It is Geometric series.

$S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$.

$a = \frac{1}{4}$; $\quad r = \frac{t_2}{t_1} = \frac{(1/4)^2}{1/4} = \frac{1}{16_4} \times \frac{4}{1}$

$r = \frac{1}{4} < 1. \quad ; n = n$

$$= \frac{\frac{1}{4}\left[1-\left(\frac{1}{4}\right)^n\right]}{1-\frac{1}{4}}$$

$$= \frac{\frac{1}{4}\left[1-\left(\frac{1}{4}\right)^n\right]}{\frac{4-1}{4}} \Rightarrow \frac{\frac{1}{4}\left[1-\left(\frac{1}{4}\right)^n\right]}{\frac{3}{4}}$$

$$= \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{3}\left[1-\left(\frac{1}{4}\right)^n\right]$$

$$= \frac{1}{3}\left[1-\left(\frac{1}{4}\right)^n\right]$$

H.p.

G.S :- Sum of G.S upto n terms

$$\boxed{\begin{aligned} S_n &= \frac{a(1-r^n)}{1-r} \quad \text{if } r < 1 \\ S_n &= \frac{a(r^n-1)}{r-1} \quad \text{if } r > 1 \\ S_n &= na. \quad \text{if } r = 1 \end{aligned}}$$

Sum of Geometric Infinity series

$$S_\infty = \frac{a}{a-r} \;\#.$$

Properties of Determinants:-

* $|A^T| = |A|$.
* $|AB| = |A| \cdot |B|$.
* $|A|^n = |A^n|$.
* $|KA| = K^n |A|$

A is Matrix
K is const.
n is order of Matrix.

* $|AA^T| = |A||A^T|$

$= |A| \cdot |A|$

$$\boxed{|A \cdot A^T| = |A|^2}$$

Example

$$A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

$$KA = 2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

$n = 2$

$$|KA| = 2^2 \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$= 4(4-6)$

$= 4(-2)$

$|KA| = -8$

$$|KA| = \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix}$$

$= 16 - 24$

$|KA| = -8$

16). If A is a Square Matrix and $|A|=2$ find the value of $|A \cdot A^T|$.

Sol.

$$|A \cdot A^T| = |A|^2$$

$$= 2^2$$

$$|A \cdot A^T| = 4.$$

17). If A and B are Square Matrix of Order '3' Such that $|A| = -1$ and $|B| = 3$. Find the value of $|3AB|$.

Sol:

$n=3$; $|A|=-1$; $|B|=3$.

$$|3AB| = 3^n |AB|$$

$$= 3^n |A| |B|$$

$$\boxed{|AB| = |A| \cdot |B|}$$

$$= 3^3 (-1)(3)$$

$$|3AB| = -81 \ \#$$

18). If $\lambda = -2$, determine the value of

$$\begin{vmatrix} 0 & 2\lambda & 1 \\ \lambda^2 & 0 & 3\lambda^2+1 \\ -1 & 6\lambda-1 & 0 \end{vmatrix}$$

Sol

$$= \begin{vmatrix} 0 & 2(-2) & 1 \\ (-2)^2 & 0 & 3(-2)^2+1 \\ -1 & 6(-2)-1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} \overset{+}{0} & \overset{-}{-4} & \overset{+}{1} \\ 4 & 0 & 13 \\ -1 & -13 & 0 \end{vmatrix}$$

$$= 0 + 4 \begin{vmatrix} 4 & 13 \\ -1 & 0 \end{vmatrix} + 1 \begin{vmatrix} 4 & 0 \\ -1 & -13 \end{vmatrix}$$

$$= 0 + 4(0-(-13))$$

$$= 0 + 4(0+13) + 1(-52+0)$$

$$= 4(13) + 1(-52)$$

$$= 52 - 52$$

$$= 0$$

#

15) without expanding, Evaluate the determinant

(i) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

Sol:

$$\Delta = \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$$

Take '3x' common from R_3

$$\Delta = 3x \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 2 & 3 & 4 \end{vmatrix} \quad R_1 \equiv R_3$$

$$\Delta = 3x(0)$$

$$\Delta = 0.$$

(ii) $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$

Sol:

$$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} \quad R_1 \to R_1 + R_2$$

Take $x+y+z$ Common from R_1

$$\Delta = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} \quad R_1 \equiv R_3$$

$$= (x+y+z)(0)$$

$$\Delta = 0$$

---

19) Determine the roots of the equation

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$$

Sol. Expand R_1

$$1\begin{vmatrix} -2 & 5 \\ 2x & 5x^2 \end{vmatrix} - 4\begin{vmatrix} 1 & 5 \\ 1 & 5x^2 \end{vmatrix} + 20\begin{vmatrix} 1 & -2 \\ 1 & 2x \end{vmatrix} = 0$$

$$1(-10x^2 - 10x) - 4(5x^2 - 5) + 20(2x+2) = 0$$

$$-10x^2 - 10x - 20x^2 + 20 + 40x + 40 = 0$$

$$-30x^2 + 30x + 60 = 0$$

$$\div -30$$

$$x^2 - x - 2 = 0.$$

$$\frac{-2}{x} \quad \frac{-2}{1} \quad \frac{1}{x} \quad -1$$

It is Q.E so factorize it

$$(x-2)(x+1) = 0$$

$$x - 2 = 0; \quad x + 1 = 0 \Rightarrow x = 2; \; x = -1$$

$$\Rightarrow x = -1, 2.$$

20). Verify that det(AB) = (det A)(det B) for

$$A = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$$

Sol:

Verify $|AB| = |A| \cdot |B|$

L.H.S: $|AB|$.

$$AB = \begin{bmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{bmatrix}$$

$$= \left[\begin{array}{ccccccccc} 1 & -2 & 9 & 3 & 4 & 7 & 3 & 0 & 5 \\ \hline 4 & 3 & -2 & 4 & 0 \; 3 & -2 & 4 & 3 & -2 \\ 1 & 0 & 7 & 1 & 0 & 7 & 1 & 0 & 7 \\ 2 & 3 & -5 & 2 & 3 & -5 & 2 & 3 & -5 \end{array}\right]$$

$$= \begin{bmatrix} 4-6-18 & 12+12-14 & 12+0-10 \\ 1+0+63 & 3+0+49 & 3+0+35 \\ 2-6-45 & 6+12-35 & 6+0-25 \end{bmatrix}$$

$$AB = \begin{bmatrix} -20 & 10 & 2 \\ 64 & 52 & 38 \\ -49 & -17 & -19 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} \overset{+}{-20} & \overset{-}{10} & \overset{+}{2} \\ 64 & 52 & 38 \\ -49 & -17 & -19 \end{vmatrix}$$

Expand R_1

$$= -20 \begin{vmatrix} 52 & 38 \\ -17 & -19 \end{vmatrix} - 10 \begin{vmatrix} 64 & 38 \\ -49 & -19 \end{vmatrix} + 2 \begin{vmatrix} 64 & 52 \\ -49 & -17 \end{vmatrix}$$

$$= -20(-988+646) - 10(-1216+1862) + 2(-1088+2548)$$

$$= -20(-342) - 10(646) + 2(1460)$$

$$= 6840 - 6460 + 2920$$

$$|AB| = 3300 \rightarrow \text{L.H.S.}$$

R.H.S:-

$$|A| = \begin{array}{c} + \qquad - \qquad + \\ \begin{vmatrix} 4 & 3 & -2 \\ 1 & 0 & 7 \\ 2 & 3 & -5 \end{vmatrix} \end{array}$$

Expand R_1

$$= 4\begin{vmatrix} 0 & 7 \\ 3 & -5 \end{vmatrix} - 3\begin{vmatrix} 1 & 7 \\ 2 & -5 \end{vmatrix} - 2\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix}$$

$$= 4(0-21) - 3(-5-14) - 2(3-0)$$

$$= 4(-21) - 3(-19) - 2(3)$$

$$= -84 + 57 - 6$$

$$|A| = -33$$

$$|B| = \begin{array}{c} + \qquad - \qquad + \\ \begin{vmatrix} 1 & 3 & 3 \\ -2 & 4 & 0 \\ 9 & 7 & 5 \end{vmatrix} \end{array} \text{(Expand } R_1\text{)}$$

$$= 1\begin{vmatrix} 4 & 0 \\ 7 & 5 \end{vmatrix} - 3\begin{vmatrix} -2 & 0 \\ 9 & 5 \end{vmatrix} + 3\begin{vmatrix} -2 & 4 \\ 9 & 7 \end{vmatrix}$$

$$= 1(20-0) - 3(-10-0) + 3(-14-36)$$

$$= 1(20) - 3(-10) + 3(-50)$$

$$= 20 + 30 - 150$$

$$|B| = -100$$

$$\therefore |A||B| = (-33)(-100)$$

$$|A||B| = +3300 \rightarrow \text{R.H.S}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\therefore |AB| = |A||B|$$

21) Using co-factor of elts of 2nd row, Evaluate $|A|$, where $|A| = \begin{bmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Sol

$$|A| = \begin{vmatrix} 5 & 3 & 8 \\ \overset{-}{2} & \overset{+}{0} & \overset{-}{1} \\ 1 & 2 & 3 \end{vmatrix}$$

Expand R_2

$$\cancel{A = -2\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} + 3\begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} - 8\begin{vmatrix} & \\ & \end{vmatrix}}$$

$$|A| = -2\begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} + 0 - 1\begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix}$$

$$= -2(9-16) - 1(10-3)$$

$$= -2(-7) - 1(7)$$

$$= 14 - 7$$

$$|A| = 7 \;\#$$

eg: 7.18

If a, b, c and x are +ve ~~integers~~ real no's then S.T
$$\begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 1 \\ (c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 1 \end{vmatrix}$$
is zero.

Sol

$$\Delta = \begin{vmatrix} (a^x+a^{-x})^2 & (a^x-a^{-x})^2 & 1 \\ (b^x+b^{-x})^2 & (b^x-b^{-x})^2 & 1 \\ (c^x+c^{-x})^2 & (c^x-c^{-x})^2 & 1 \end{vmatrix}$$

$$(a^x+a^{-x})^2 = \Big(\underset{a}{a^x} + \underset{b}{\frac{1}{a^x}}\Big)^2$$

$$= (a^x)^2 + 2\cancel{(a^x)}\left(\frac{1}{\cancel{a^x}}\right) + \left(\frac{1}{a^x}\right)^2$$

$$(a^x+a^{-x})^2 = a^{2x} + 2 + \frac{1}{a^{2x}}$$

$$\text{|||ly } (a^x-a^{-x})^2 = a^{2x} - 2 + \frac{1}{a^{2x}}$$

$$(a^x+a^{-x})^2 = \cancel{a^{2x}} + 2 + \cancel{\frac{1}{a^{2x}}}$$

$$\overset{(-)}{}(a^x-a^{-x})^2 = \overset{(-)}{\cancel{a^{2x}}} \overset{(+)}{-2} \overset{(-)}{+} \cancel{\frac{1}{a^{2x}}}$$

$$(a^x+a^{-x})^2-(a^x-a^{-x})^2 = 4 \;; \; (b^x+b^{-x})^2-(b^x-b^{-x})^2 = 4$$

$$(c^x+c^{-x})^2-(c^x-c^{-x})^2 = 4$$

$$\Delta = \begin{vmatrix} 4 & (a^x-a^{-x})^2 & 1 \\ 4 & (b^x-b^{-x})^2 & 1 \\ 4 & (c^x-c^{-x})^2 & 1 \end{vmatrix} \quad C_1 \to C_1 - C_2$$

Take 4 as common from C_1

$$\Delta = 4\begin{vmatrix} 1 & (a^x - a^{-x})^2 & 1 \\ 1 & (b^x - b^{-x})^2 & 1 \\ 1 & (c^x - c^{-x})^2 & 1 \end{vmatrix} \quad C_1 \equiv C_3$$

$$\Delta = 4(0)$$

$$\Delta = 0$$

eg: 7.19

If $A = \begin{bmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{bmatrix}$ & $B = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

without expanding det, S.T $|B| = 2|A|$.

Sol.

$$|B| = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} \quad R_1 \to R_1 + R_2 + R_3$$

$$= 2\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}$$

$$= 2\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ -b & -c & -a \\ -c & -a & -b \end{vmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$= 2\begin{vmatrix} a & b & c \\ -b & -c & -a \\ -c & -a & -b \end{vmatrix} \quad R_1 \to R_1 + R_2 + R_3$$

Take -1 as common from R_2 & R_3

$$= 2(-1)(-1)\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$|B| = 2|A|.$$

eg: 7.20 Evaluate $\begin{vmatrix} 2014 & 2017 & 0 \\ 2020 & 2023 & 1 \\ 2023 & 2026 & 0 \end{vmatrix}$

Sol:-

$$\Delta = \begin{vmatrix} 2014 & 3 & 0 \\ 2020 & 3 & 1 \\ 2023 & 3 & 0 \end{vmatrix} \quad C_2 \to C_2 - C_1$$

$$= 3\begin{vmatrix} 2014 & 1 & \overset{+}{0} \\ 2020 & 1 & \overset{-}{1} \\ 2023 & 1 & 0^{+} \end{vmatrix} \quad \text{Expand } C_3$$

$$= 3\left[0 - 1\begin{vmatrix} 2014 & 1 \\ 2023 & 1 \end{vmatrix} + 0\right]$$

$$= 3\left[-1(2014 - 2023)\right]$$

$$= 3\left[-1(-9)\right]$$

$$= 27.$$

eg: 7.21: Find value of x if $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$

Sol:

Since it is a ~~lo~~ upper triangular Matrix, the value of determinant is product of leading diagonal

$\therefore (x-1)(x-2)(x-3) = 0 \Rightarrow x-1=0;\ x-2=0;\ x-3=0$

$\Rightarrow x=1;\ x=2;\ x=3$

$\therefore x = 1, 2, 3.$

Eg: 7.22

P.T $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

Sol.

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix} \quad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ (x-y)(x+y) & (y-z)(y+z) & z^2 \end{vmatrix}$$

Take $(x-y), (y-z)$ common from C_1 & C_2

$$= (x-y)(y-z) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ x+y & y+z & z^2 \end{vmatrix} \quad \text{Expand } R_1$$

$$= (x-y)(y-z) \left[0 - 0 + 1 \begin{vmatrix} 1 & 1 \\ x+y & y+z \end{vmatrix} \right]$$

$$= (x-y)(y-z) \left[1 \{ (y+z) - (x+y) \} \right]$$

$$= (x-y)(y-z) \left[\cancel{y} + z - x - \cancel{y} \right]$$

$$= (x-y)(y-z)(z-x)_{\#}$$