தமிழ்நாடு பள்ளிக்கல்வித் துறை

மதுரை மாவட்டம்

12 – கணிதம்

சிறப்பு பயிற்சி கையேடு 2021-22

வழிகாட்டல்	மேற்பார்வை
உயர்திரு. இரா.சுவாமிநாதன் அவர்கள் முதன்மைக் கல்வி அலுவலர் மதுரை மாவட்டம்	திரு.ப. குமரேசன் அவர்கள் தலைமை ஆசிரியர் அரசு மாதிரி மேல்நிலைப்பள்ளி P. அம்மாபட்டி.

ஆசிரியர் குழு:—

ஒருங்கிணைப்பாளர்கள்:	
1. C. தமிழ்குமரன் முதுகலை ஆசிரியர் அல்அமீன், மேல்நிலைப்பள்ளி புதூர், மதுரை.	2. D. பாண்டியன் முதுகலை ஆசிரியர் காந்திஜி நினைவு அரசு ஆண்கள் மேல்நிலைப்பள்ளி பேரையூர்.
P. செந்தில் குமார், முதுகலை ஆசிரியர் அரசு மேல்நிலைப்பள்ளி, உறங்கான்பட்டி, மதுரை	R. தமிழ்ச்செல்வி, முதுகலை ஆசிரியர் PKN மேல்நிலைப்பள்ளி, திருமங்கலம்
R. விஜயகுமார், முதுகலை ஆசிரியர் TVS மேல்நிலைப்பள்ளி, மதுரை.	M. சங்கரமூர்த்தி, ஸ்ரீ கந்தசாமி வித்யாலயம் மேல்நிலைப்பள்ளி, பாப்புநாயக்கன்பட்டி.
K.S. லதா, முதுகலை ஆசிரியர் அரசு மேல்நிலைப்பள்ளி, அவனியாபுரம்	S. முருகேஸ்வரி, முதுகலை ஆசிரியர், அரசு மேல்நிலைப்பள்ளி,T. கல்லுப்பட்டி
D. விவேகி, முதுகலை ஆசிரியர் VHN மேல்நிலைப்பள்ளி, மதுரை.	K.R.P. நித்யா தேவி, முதுகலை ஆசிரியர் அரசு பெண்கள் மேல்நிலைப்பள்ளி, கருங்காலக்குடி
V. விஜயபாபு, முதுகலை ஆசிரியர் அரசு மேல்நிலைப்பள்ளி, M. சுப்புலாபுரம்	N. இராமகிருஷ்ணன், முதுகலை ஆசிரியர், அரசு ஆண்கள் மேல்நிலைப்பள்ளி, எழுமலை
S. திலீப் குமார், முதுகலை ஆசிரியர் அரசு ஆண்கள் மேல்நிலைப்பள்ளி, கருங்காலக்குடி	M. சுபாஷினி, முதுகலை ஆசிரியர் அரசு மேல்நிலைப்பள்ளி, சத்திரப்பட்டி

12 கணிதம் — சிறப்பு பயிற்சி கையேடு

- ❖ மாணவர்களுக்கு எளிமையாகப் புரியும் வகையில் முக்கிய வினாக்களுக்கு, விடைகள் மிகவும் எளிய முறையில் விளக்கப்பட்டுள்ளது.
- ❖ மாணவர்கள் இந்தக் கையேட்டில் உள்ள கணக்குகளை புரிந்து படிப்பதுடன் மீண்டும், மீண்டும் எழுதிப்பார்ப்பது அவசியம்.
- ❖ பாடப்புத்தகத்தில் இடம்பெற்றுள்ள அனைத்து ஒரு மதிப்பெண் வினாக்களில் நன்கு பயிற்சி மேற்கொண்டால் 14 மதிப்பெண்கள் எளிதாகப் பெறலாம்
- ❖ தேர்வில், வினாவினை பலமுறை படித்து, கேள்வியைப் புரிந்து, விடையைக் கண்டறிந்த பின்பு விடையளிக்க வேண்டும். முழுமையான விடை தெரியவில்லை என்றாலும், அந்த வினாவிற்கு விடையை உங்களுக்கு தெரிந்த அளவிற்கு பதில் அளித்தால் நிறைய மதிப்பெண்களைப் (Step Mark) பெறலாம்.
- ❖ மாணவர்கள் எந்தக் கேள்வியையும் விட்டுவிடாமல் கண்டிப்பாக அனைத்து கேள்விகளுக்கும் உங்களுக்கு தெரிந்த அளவில் விடையளிக்க வேண்டும்.
- ❖ தேவையான இடங்களில் படங்கள் வரையவேண்டும். சூத்திரங்கள் எழுத வேண்டும். இவற்றிற்கு மதிப்பெண்கள் உண்டு.
- ❖ தன்னம்பிக்கையோடு கணிதத் தேர்வினை எதிர்கொள்ள வேண்டும்.

முயற்சி + பயிற்சி = வெற்றி

1. Application of Matrix and Determinants

Important points:

1. $A^{-1} = \frac{1}{|A|} adjA$
2. $A^{-1} = \pm \frac{1}{\sqrt{|adjA|}} adjA$
3. $A = \pm \frac{1}{\sqrt{|adjA|}} adj\ (adjA)$
4. The rank of a matrix is equal to the number of non-zero rows in row-echelon form of the matrix
5. Matrix inversion method: A X = B ==> $X = A^{-1}B$
6. Cramer's method:

   $$x = \frac{\Delta_1}{\Delta},\ y = \frac{\Delta_2}{\Delta}\ ,\ z = \frac{\Delta_3}{\Delta}$$
7. If $AA^T = A^TA = I$ then A is orthogonal

CHAPTER 1

Application of Matrices and Determinants:

2, 3 Mark Questions:

1. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$

Solution:

$$|A| = \begin{vmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{vmatrix} = 2\,(9 - 2) + 1\,(-15 + 3) + 3\,(-10 + 9)$$

$$= 2(7)+1(-12)+3(-1)=14-12-3= -1 \neq 0$$

$$\begin{bmatrix} \cancel{2} & \cancel{-1} & \cancel{3} & \cancel{2} & \cancel{-1} & \cancel{3} \\ \cancel{-5} & 3 & 1 & -5 & 3 & \cancel{1} \\ \cancel{-3} & 2 & 3 & -3 & 2 & \cancel{3} \\ \cancel{2} & -1 & 3 & 2 & -1 & \cancel{3} \\ \cancel{-5} & 3 & 1 & -5 & 3 & \cancel{1} \\ \cancel{-3} & \cancel{2} & \cancel{3} & \cancel{-3} & \cancel{2} & \cancel{3} \end{bmatrix}$$

$$\text{adj A} = \begin{bmatrix} 9-2 & -3+15 & -10+9 \\ 6+3 & 6+9 & 3-4 \\ -1-9 & -15-2 & 6-5 \end{bmatrix}^T$$

$$\text{adjA} = \begin{bmatrix} 7 & 12 & -1 \\ 9 & 15 & -1 \\ -10 & -17 & 1 \end{bmatrix}^T = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj A} = \frac{1}{-1}\begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$$

2. If A= $\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that A (adj A) = (adj A) A= $|A|I_2$

Solution :

adj A= $\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$|A|$ = $\begin{vmatrix} 3 & 4 \\ 5 & 8 \end{vmatrix}$ = 24 – 20 = 4

$|A|I_2$ = 4 $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ……….. (1)

A (adj A) = $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ……………… (2)

(adj A) A= $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ ………………..(3)

From (1), (2) and (3)

A (adj A) = (adj A) A = $|A|I_2$

3. If A= $\begin{bmatrix}0 & -3\\1 & 4\end{bmatrix}$ and B= $\begin{bmatrix}-2 & -3\\0 & -1\end{bmatrix}$,verify that $(AB)^{-1}$ = $B^{-1}A^{-1}$

Solution:

A B = $\begin{bmatrix}0 & -3\\1 & 4\end{bmatrix}\begin{bmatrix}-2 & -3\\0 & -1\end{bmatrix} = \begin{bmatrix}0+0 & 0+3\\-2+0 & -3-4\end{bmatrix} = \begin{bmatrix}0 & 3\\-2 & -7\end{bmatrix}$

$|AB|$ = 0 + 6 = 6; adj AB= $\begin{bmatrix}-7 & -3\\2 & 0\end{bmatrix}$

$(AB)^{-1}$ = $\frac{1}{|AB|}$ adj (AB) = $\frac{1}{6}\begin{bmatrix}-7 & -3\\2 & 0\end{bmatrix}$ ……… (1)

$B^{-1}A^{-1}$ = $\frac{1}{6}\begin{bmatrix}-7 & -3\\2 & 0\end{bmatrix}$ ………………… (2)

From (1) and (2)

$(AB)^{-1}$ = $B^{-1}A^{-1}$

4. If A= $\begin{bmatrix}3 & 2\\7 & 5\end{bmatrix}$ and B= $\begin{bmatrix}-1 & -3\\5 & 2\end{bmatrix}$ verify that $(AB)^{-1}$ = $B^{-1}A^{-1}$

Solution:

A B = $\begin{bmatrix}3 & 2\\7 & 5\end{bmatrix}\begin{bmatrix}-1 & -3\\5 & 2\end{bmatrix} = \begin{bmatrix}-3+10 & -9+4\\-7+25 & -21+10\end{bmatrix} = \begin{bmatrix}7 & -5\\18 & -11\end{bmatrix}$

$|AB|$ = -77 + 90 = 13; adj (AB) = $\begin{bmatrix}-11 & 5\\-18 & 7\end{bmatrix}$

$(AB)^{-1}$ = $\frac{1}{|AB|}$ adj (AB) = $\frac{1}{13}$= $\begin{bmatrix}-11 & 5\\-18 & 7\end{bmatrix}$ ……… (1)

$B^{-1}A^{-1}$ = $\frac{1}{13}\begin{bmatrix}-11 & 5\\-18 & 7\end{bmatrix}$ ………………… (2)

From (1) and (2)

$(AB)^{-1}$ = $B^{-1}A^{-1}$

5. Find a matrix A if adjA = $\begin{bmatrix}2 & -4 & 2\\-3 & 12 & -7\\-2 & 0 & 2\end{bmatrix}$

Solution:

$A = \pm \frac{1}{\sqrt{|adjA|}}$ adj (adj A)

$|adjA|$ = 2 (24 -0) +4 (-6 -14) +2 (0 + 24)

= 48 – 80 + 48 = 16

$\sqrt{|adjA|}$ = $\sqrt{16}$ = 4

$$\text{adj (adj A)} = \begin{bmatrix} 24-0 & -14+6 & 0+24 \\ 0+8 & 4+4 & 8-0 \\ 28-24 & -6+14 & 24-12 \end{bmatrix}^T \quad \left| \begin{array}{cccccc} \cancel{2} & -\cancel{4} & \cancel{2} & \cancel{2} & -\cancel{4} & \cancel{2} \\ -\cancel{3} & 12 & -7 & -3 & 12 & -\cancel{7} \\ -\cancel{2} & 2 & 2 & -2 & 0 & \cancel{2} \\ \cancel{2} & 0 & 2 & 2 & -4 & \cancel{2} \\ -\cancel{3} & 12 & -7 & -3 & 12 & -\cancel{7} \\ -\cancel{2} & \cancel{0} & \cancel{2} & -\cancel{2} & \cancel{0} & \cancel{2} \end{array} \right.$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$A = \pm \frac{1}{\sqrt{|adjA|}} \text{ adj (adj A)} = \pm \frac{1}{4} \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix} = \pm \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

6. If $A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$ Prove that $A^{-1} = A^T$

Solution:

$AA^T = I \Rightarrow A^T = A^{-1}$

$$A^T = \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$AA^T = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix} \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} = \frac{1}{81} \begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$A^{-1} = A^T$$

7. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$ show that $(F(\alpha))^{-1}$ = F(-α)

Solution:

$$F(-\alpha) = \begin{bmatrix} \cos(-\alpha) & 0 & \sin(-\alpha) \\ 0 & 1 & 0 \\ -\sin(-\alpha) & 0 & \cos(-\alpha) \end{bmatrix}$$

$$F(-\alpha) = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \text{........ (1)}$$

$$(F(\alpha))^{-1} = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad \text{........ (2)}$$

From (1) and (2), $(F(\alpha))^{-1}$ = F(-α)

8. If adj (A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}

Solution:

$|adjA| = 0$ (12 -0) +2 (36 -18) + 0 (0 +6) =36

$\sqrt{|adjA|} = \sqrt{36} = 6$

$A^{-1} = \pm \frac{1}{|A|} adjA$

$= \pm \frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & -2 & 0 \\ -3 & 0 & 6 \end{bmatrix}$

9. If adj A = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & -2 & 0 \\ -3 & 0 & 6 \end{bmatrix}, find A^{-1}$

Solution:

$|adjA| = -1(1-4) - 2(1-4) + 2(2-2) = 3 + 6 + 0 = 9$

$\sqrt{|adjA|} = \sqrt{9} = 3$

$A^{-1} = \pm \frac{1}{\sqrt{|adjA|}} adjA = \pm \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

10. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

Solution

$$|A| = 14 - 9 = 5$$

$$adjA = \begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$$

$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{5}\begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix}$

$(A^{-1})^T = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$ …………………….. (1)

$(A^T)^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$ …………………….. (2)

From (1) and (2) $(A^{-1})^T = (A^T)^{-1}$

11. Prove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal

Solution

If $AA^T = A^TA = I$ then A is orthogonal

$A = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

$AA^T = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$

$= \begin{bmatrix} cos^2\theta + sin^2\theta & Cos\theta sin\theta - sin\theta \cos\theta \\ sin\theta cos\theta - cos\theta sin\theta & cos^2\theta + sin^2\theta \end{bmatrix}$

$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad similarly \quad A^TA = I$

∴ A is orthogonal.

12. Find the matrix A for which $A \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$

solution:

$AB = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} B^{-1}$

$|B| = \begin{vmatrix} 5 & 3 \\ -1 & -2 \end{vmatrix} = -10 + 3 = -7$; $adjB = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$B^{-1} = \frac{1}{|B|}$ adj B $= -\frac{1}{7} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$

$A = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} (-\frac{1}{7}) \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} = -\frac{1}{7} \begin{bmatrix} -28 & +7 & -42 & +35 \\ -14 & +7 & -21 & +35 \end{bmatrix}$

$A = -\frac{1}{7} \begin{bmatrix} -21 & -7 \\ -7 & 14 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}$

13. Find the rank of the matrix $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ by minor method

Solution:

Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ Then A is a matrix of order 3 x 3

$P(A) \leq \min(3, 3) = 3$

$|A| = \begin{vmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{vmatrix} = 3 (6 - 6) - 2 (6 – 6) - 5 (3 – 3) = 0$

$\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 – 2 = 1 \neq 0$

$\therefore P(A) = 2$

14. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form:

Solution:

The rank of a matrix is equal to the number of non-zero rows in a row –echelon form of the matrix.

$$A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix} \sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 8 & -13 & -2 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to 2R_2 + 3R_1}$$

$$\sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 0 & -45 & -30 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 4R_2} \sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 0 & 3 & 2 \end{bmatrix} R_3 \to R_3 \div (-15)$$

$\therefore P(A) = 3$

15. Find the rank of the matrix = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ by minor method

Solution:

Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ p(A) ≤ min (3, 3) = 3

$|A|$ = $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ = 1 (-4 + 6) +2 (-2 + 30) +3 (2 – 20)

= 2 + 56 – 54 = 4

$\therefore P(A) = 3$

16. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ by reducing it to an echelon form:

Solution:

The rank of a matrix is equal to the number of non-zero rows in a row-echelon form of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & 4 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 2R_1}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_2}$$

P(A) = 2

17. $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ show that $A^2 - 3A - 7I_2 = 0$ Hence find A^{-1}.

Solution:

$$A^2 = AA = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$A^2 - 3A - 7I_2 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3-0 & 1+6-7 \end{bmatrix}$$

$$|A| = -10+3 = -7$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0_2$$

$$\text{adj } A = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$

18. Solve $2x - y = 8$; $3x + 2y = -2$ Using matrix inversion method

Solution:

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

AX=B ==> $X = A^{-1}B$

$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 4 + 3 = 7 \neq 0$; adj $A = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

X = A-1B

$$= \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

$$= \frac{1}{7}\begin{bmatrix} 16-2 \\ -24-4 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -28 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$\therefore$ x = 2, y = -4

19. Solve $\frac{3}{x} + 2y = 12$, $\frac{2}{x} + 3y = 13$ by Cramer's rule

Solution:

$$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$$

$$\Delta_1 = \begin{vmatrix} 12 & 2 \\ 13 & 3 \end{vmatrix} = 36 - 26 = 10$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 \\ 2 & 13 \end{vmatrix} = 39 - 24 = 15$$

$$\frac{1}{x} = \frac{\Delta_1}{\Delta} = \frac{10}{5} = 2 \Rightarrow x = \frac{1}{2}$$

$$y = \frac{\Delta_2}{\Delta} = \frac{15}{5} = 3$$

$\therefore x = \frac{1}{2}$, y = 3

20. Solve 5x – 2y + 16 = 0, x + 3y – 7 = 0 by Cramer's rule

Solution:

5x – 2y = -16, x + 3y = 7

$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix}$ = 15 + 2 = 17

$\Delta_1 = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix}$ = -48 +14 = -34

$\Delta_2 = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix}$ = 35 +16 = 51

$x = \frac{\Delta_1}{\Delta} = \frac{-34}{17} = -2$

$y = \frac{\Delta_2}{\Delta} = \frac{51}{17} = 3$

$\therefore x = -2$, y = 3

5 Marks questions:

1. If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ verify that A(adj A) = (adj A) A = $|A|I_3$

Solution:

$|A| = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ = 8(21-16) + 6(-18 + 8) +2(24-14)

= 40- 60 + 20 = 0

$$\begin{array}{cccccc} 8 & -6 & 2 & 8 & -6 & 2 \\ -6 & 7 & -4 & -6 & 7 & -4 \\ 2 & -4 & 3 & 2 & -4 & 3 \\ 8 & -6 & 2 & 8 & -6 & 2 \\ -6 & 7 & -4 & -6 & 7 & -4 \\ 2 & -4 & 3 & 2 & -4 & 3 \end{array}$$

$|A|I_3 = 0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ………… (1)

$adjA = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

$A(adjA) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ………… (2)

$(adjA)A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ………… (3)

From (1), (2) and (3)

A(adjA) = (adjA)A = $|A|I_3$

2. $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ find the products AB and BA and hence solve the system of equations x – y + z = 4, x – 2y – 2z = 9, 2x + y +3z = 1

Solution:

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8I$$

$$BA = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8I$$

AB = BA = 8I

AB = 8I $\Rightarrow \frac{1}{8}A = B^{-1}$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$B\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -16 & +36 & +4 \\ -28 & +9 & +3 \\ 20 & -27 & -1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

∴ x= 3, y = -2, z = -1

3. Solve 2x + 3y – z = 9, x + y + z = 9 , 3x – y – z = -1 using inversion method:

Solution:

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$$

AX = B

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 2(-1 +1) - 3(-1-3) -1(-1-3)$$

$$\begin{array}{cccccc} 2 & 3 & 1 & 2 & 3 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 3 & -1 & -1 \\ 2 & 3 & 1 & 2 & 3 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 3 & -1 & -1 \end{array}$$

$$= 0 + 12 + 4 = 16 \neq 0$$

$$adjA = \begin{bmatrix} -1+1 & 3+1 & -1-3 \\ 1+3 & -2+3 & 9+2 \\ 3+1 & -1-2 & 2-3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 4 & -4 \\ 4 & 1 & 11 \\ 4 & -3 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA$$

$$= \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$X = A^{-1}B$

$$= \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 0 & +\ 36 & -\ 4 \\ 36 & +\ 9 & +\ 3 \\ -36 & +\ 99 & +\ 1 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 32 \\ 48 \\ 64 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

∴ x = 2, y = 3, z = 4

4. Solve $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0$, $\frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0$, $\frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$ **by Cramer's rule**

Solution:

$$\Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix} = 3(-8 + 5) + 4(-4 -2) -2(-5-4)$$

= -9 -24 + 18 = -15

$$\Delta_1 = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix} = 1(-8 +5) + 4(-8 +1) - 2(-10 +2)$$

= -3-28 +16 = -15

$$\Delta_2 = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix} = 3(-8+1) - 1(-4-2) -2(-1-4)$$

$= -21 + 6 + 10 = -5$

$$\Delta_3 = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix} = 3(-2+10) + 4(-1-4) +1(-5-4)$$

$= 24 - 20 - 9 = -5$

$\frac{1}{x} = \frac{\Delta_1}{\Delta} = \frac{-15}{-15} = 1 \Longrightarrow x = 1$

$\frac{1}{y} = \frac{\Delta_2}{\Delta} = \frac{-5}{-15} = \frac{1}{3} \Longrightarrow y = 3$

$\frac{1}{z} = \frac{\Delta_3}{\Delta} = \frac{-5}{-15} = \frac{1}{3} \Longrightarrow z = 3$

$\therefore$ x = 1, y = 3, z = 3

5. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule to solve the problem.

Solution:

$\frac{1}{x} + \frac{1}{y} = \frac{1}{10}$

$\frac{1}{x} - \frac{1}{y} = \frac{1}{30}$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1-1 = -2$$

$$\Delta_1 = \begin{vmatrix} \frac{1}{10} & 1 \\ \frac{1}{30} & -1 \end{vmatrix} = -\frac{1}{10} - \frac{1}{30} = \frac{-3-1}{30} = \frac{-4}{30} = \frac{-2}{15}$$

$$\Delta_2 = \begin{vmatrix} 1 & \frac{1}{10} \\ 1 & \frac{1}{30} \end{vmatrix} = \frac{1}{30} - \frac{1}{10} = \frac{1-3}{30} = \frac{-2}{30} = \frac{-1}{15}$$

$\frac{1}{x} = \frac{\Delta_1}{\Delta} = \frac{-2/15}{-2} = \frac{-2}{15} \times \frac{1}{-2} = \frac{1}{15} \Longrightarrow x = 15$

$\frac{1}{y} = \frac{\Delta_2}{\Delta} = \frac{-1/15}{-2} = \frac{-1}{15} \times \frac{1}{-2} = \frac{1}{30} \Longrightarrow y = 30$

$\therefore$x=15minutes y = 30minutes

6. In a T-20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is y= $ax^2 + bx + c$ with respect a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20,16), (40, 22) can you conclude that the team won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70,0))

Solution:

$y = ax^2 + bx + c$

$100a + 10b + c = 8$

$400a + 20b + c = 16$

$1600a + 40b + c = 22$

$$\Delta = \begin{vmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 1600 & 40 & 1 \end{vmatrix} = 1000 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 16 & 4 & 1 \end{vmatrix}$$

= 1000 [(1(2-4) -1(4-16) +1(16-32)]

= 1000 [-2+12-16] = 1000(-6)

Δ = -6000

$$\Delta_1 = \begin{vmatrix} 8 & 10 & 1 \\ 16 & 20 & 1 \\ 22 & 40 & 1 \end{vmatrix} = 20 \begin{vmatrix} 4 & 1 & 1 \\ 8 & 2 & 1 \\ 11 & 4 & 1 \end{vmatrix}$$

= 20 [4(2-4) – 1(8 -11) +1(32-22)]

= 20 [-8+3+10] = 1000(-6)

Δ_1 = 100

$$\Delta_2 = \begin{vmatrix} 100 & 8 & 1 \\ 400 & 16 & 1 \\ 1600 & 22 & 1 \end{vmatrix} = 200 \begin{vmatrix} 1 & 4 & 1 \\ 4 & 8 & 1 \\ 16 & 11 & 1 \end{vmatrix}$$

= 200[1(8 -11) – 4(4-16) +1(44-128)]

= 200(-3+48-84)

Δ_2 = -7800

$$\Delta_3 = \begin{vmatrix} 100 & 10 & 8 \\ 400 & 20 & 16 \\ 1600 & 40 & 22 \end{vmatrix} = 2000 \begin{vmatrix} 1 & 1 & 4 \\ 4 & 2 & 81 \\ 16 & 4 & 11 \end{vmatrix}$$

= 2000 [1(22-32) – 1(44-128) +4(16-32)]

= 2000[-10+84-64]

Δ_3 = -20000

$a = \frac{\Delta_1}{\Delta} = \frac{100}{-6000} = \frac{-1}{60}$

$b = \frac{\Delta_2}{\Delta} = \frac{-7800}{-6000} = \frac{13}{10}$

$c = \frac{\Delta_2}{\Delta} = \frac{20000}{-6000} = \frac{-10}{3}$

$y = \frac{-1}{60}x^2 + \frac{13}{10}x - \frac{10}{3}$

245 85

$x = 70 \Rightarrow y = \frac{-1}{\not{60}} \times \not{4900} + \frac{13}{10} \times 70 - \frac{10}{3} = -\frac{\not{255}}{\not{3}} + 91$

3

y = 6 metres

$\therefore$The ball went for a super six. The team won the match

7. The upward speed (t) of a rocket at time t is approximated by v(t) = at^2 + bt + c, $0 \le t \le 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t=6 and t=9 seconds are respectively 64, 133 and 208 miles per seconds. Find the speed at time t = 15 seconds (Use Gaussian elimination method):

Solution:

V(t) = c + bt + at^2

V(3) = 64 $\Rightarrow c + 3b + 9a = 64$

V(6) = 133 $\Rightarrow$ c + 6b + 36a = 133

V(9) = 208 $\Rightarrow$ c+ 9b + 81a = 208

$$[A|\,B] = \left[\left[\begin{array}{ccc|c} 1 & 3 & 9 & 64 \\ 1 & 6 & 36 & 133 \\ 1 & 9 & 81 & 208 \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 3 & 9 & 64 \\ 0 & 3 & 27 & 69 \\ 0 & 6 & 72 & 144 \end{array}\right]\right]$$

$$\xrightarrow[R_3 \to R_3 \div 6]{R_2 \to R_2 \div 3} \left[\begin{array}{ccc|c} 1 & 3 & 9 & 64 \\ 0 & 1 & 9 & 23 \\ 0 & 1 & 12 & 24 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 3 & 9 & 64 \\ 0 & 1 & 9 & 23 \\ 0 & 0 & 3 & 1 \end{array}\right]$$

3a = 1 $\Rightarrow$ a = $\frac{1}{3}$

b +9a = 23$\Rightarrow$b + 9 (1/3) = 23 $\Rightarrow$ b = 23 -3 = 20

c + 3b + 9a =64 $\Rightarrow$ c +3 (20) + 9(1/3) = 64

$\Rightarrow$ c =64 – 60-3 = 1

V(t) = 1/3 t^2 + 20 t+1

V(15) = 1/3 (225) + 20 x15 + 1 =75 + 300 + 1= 376 m/s

8. A boy is walking along the path $y = ax^2 + 6x + c$ through the points (-6, 8), (-2, -12) and (3, 8). He wants to meet his friend? (Use Gaussian elimination method)

Solution:

$y = c + bx + ax^2$

c- 6b + 36a = 8

c – 2b + 4a = -12

c + 3b + 9a = 8

$$[A\,|B] = \left[\begin{array}{ccc|c} 1 & -6 & 36 & 8 \\ 1 & -2 & 4 & -12 \\ 1 & 3 & 9 & 8 \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & -6 & 36 & 8 \\ 0 & 4 & -32 & -20 \\ 0 & 9 & -27 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 \div 9]{R_2 \to R_2 \div 4} \left[\begin{array}{ccc|c} 1 & -6 & 36 & 8 \\ 0 & 1 & -8 & -5 \\ 0 & 1 & -3 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -6 & 36 & 8 \\ 0 & 1 & -8 & -5 \\ 0 & 0 & 5 & 5 \end{array}\right]$$

5a = 5 $\Rightarrow$ a = $\frac{5}{5}$ = 1

b – 8a = -5 $\Rightarrow$ b-8 = -5 $\Rightarrow$ $b = -5 + 8 = 3$

c – 6b + 36 = 8 $\Rightarrow$ c – 18 + 36 = 8 $\Rightarrow$c = 8 + 18 – 36 = -10

$y = x^2 + 3x - 10$

x = 7 $\Rightarrow$ y = 49 + 21 – 10 = 60

He will meet his friend

9) If $ax^2 + bx + c$ is divided by x + 3, x – 5 and x – 1, the remainders are 21, 61 and 9 respectively. Find a, b and c (Use Gaussian elimination method).

Solution: p(x) = c + bx + ax^2

P(-3) = 21 $\Rightarrow$ c – 3b + 9a = 21

P(5) = 61 $\Rightarrow$ c + 5b + 25a = 61

P(1) = 9 $\Rightarrow$ c + b + a = 9

$$[A|\,B] = \left[\begin{array}{ccc|c} 1 & -3 & 9 & 21 \\ 1 & 5 & 25 & 61 \\ 1 & 1 & 1 & 9 \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & -3 & 9 & 21 \\ 0 & 8 & 16 & 40 \\ 0 & 4 & -8 & -12 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 \div 4]{R_2 \to R_2 \div 8} \left[\begin{array}{ccc|c} 1 & -3 & 9 & 21 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & -2 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & -3 & 9 & 21 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -8 \end{array}\right]$$

$-4a = -8$ $\Rightarrow$ a = $\frac{-8}{-4}$ =2

$b + 2a = 5 \Rightarrow b + 4 = 5 \Rightarrow b = 5 - 4 = 1$

$c - 3b + 9a = 21 \Rightarrow c - 3 + 18 = 21 \Rightarrow c = 21 + 3 - 18 = 6$

$\therefore a = 2, b = 1, c = 6$

CHAPTER - 2

COMPLEX NUMBERS

Important Hints:

- $i^2 = -1, \qquad i^3 = -i\ , \qquad i^4 = 1$
- Rectangular form of a complex number is x + iy real part is x, Imaginary part is y.
- The conjugate of the complex number z= x + iy is x –iy and is denoted by $\overline{z}$
- If z = x + iy then modulus of z is $|z| = \sqrt{x^2 + y^2}$
- Triangle inequality:
  For any two complex number $z_1 and z_2, |z_1 + z_2| \le |z_1|$ + $|z_1|$
- $\sqrt{a+ib} = \pm\left[\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right]$
- Additive inverse of z is –z multiplicative inverse of z is 1/z
- z is real if any only if z = $\overline{z}$ and z is purely imaginary if and only if z = - $\overline{z}$
- Distance between two complex numbers, z_1 and z_2 is $|z_1 - z_2|$
- $|z - z_0| = r$is the complex form of the equation of a circle.
  $centre is z_0$ and radius is r.
- $\sqrt{a+ib} = \pm\left[\sqrt{\frac{Modulus+R.P}{2}} + {}^{i}\sqrt{\frac{Modulus-R.P}{2}}\right]$
- $\sqrt{a-ib} = \pm\left[\sqrt{\frac{Modulus+R.P}{2}} - {}^{i}\sqrt{\frac{Modulus-R.P}{2}}\right]$

Two Marks Questions:

1) i) $i^{1948} - i^{-1869} = i^{1948} - i^{-1868}\ .\ i^{-1}$

$= 1 - (1)\frac{1}{i} = 1 - \frac{i}{i^2} = 1 + i$

ii) $i^{59} + \frac{1}{i^{59}} = i^{56}.i^3 + \frac{1}{i^{56}.i^3} = i^3 + \frac{1}{i^3} = -i + \frac{1}{-i}$

$= -i + \frac{1xi}{-ixi} = -i + (+i) = 0$

iii) $\sum_{n=1}^{10} i^{n+50} = i^{51} + i^{52} + i^{53} + \cdots .. + i^{60}$ $\begin{cases} \therefore i^{51} + i^{52} + i^{53} + i^{54} = 0 \\ i^{55} + i^{56} + i^{57} + i^{58} = 0 \end{cases}$

$= i^{59} + i^{60} = i^3 + i^{60}$

$= i^3 + 1 = -i + 1$

$= 1 - i$

Do it yourself:

Simplify: (i) $i^{-1924} + i^{2018}$ ii) $\sum_{n=1}^{10} i^n$ $iii)$ $i, i^2, i^3 \ldots\ldots i^{40}$

2) If z= 5- 2i, w = -1+3i find i) z –iw ii) $z^2 + 2zw + w^2$

i) z – iw = 5 – 2i –i(-1 + 3i)

= 5 – 2i + i - $3i^2 = 8 - i$

ii) $z^2 + 2zw + w^2 = (z + w)^2 = [(5 - 2i) + (-1 + 3i)] = (4 + i)^2$

= $4^2 + i^2 + 8i = 15 + 8i$

Do it yourself:

(i) zw (ii) 2z + 3w

If $z_1 = 1 - 3i, z_2 = -4i, z_3 = 5$ $prove$: $(z_1 z_2)z_3 = z_1(z_2 z_3)$

LHS = $(z_1 z_2)z_3$ = { (1 – 3i) (-4i)} 5= (-4i+12i^2)5

-20i+60i^2= -60 – 20i

RHS = $z_1(z_2 z_3) = (1 - 3i)\{(-4i)5\}$= (1 – 3i) (-20i)

= -20i + $60i^2 = -60 - 20i$

Do it yourself:

$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

4) Write in rectangular form:

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$

$\frac{1+i}{1-i}$ x $\frac{1+i}{1+i} = \frac{(1+i)^2}{1^2-i^2} = \frac{1+i^2+2i}{2} = \frac{2i}{2} =$ i

$\frac{1-i}{1+i}$ x $\frac{1-i}{1-i} = \frac{(1-i)^2}{1^2-i^2} = \frac{1+i^2-2i}{2} = \frac{-2i}{2} =$ -i

$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ = $(i)^3 - (-i)^3 = i^3 + i^3 = 2i^3 = -2i$

5) If z = (2 + 3i) (1 – i) find z^{-1}

Z= (2 + 3i) (1 – i) = 2 – 2i + 3i - $3i^2 = 5 + i$

$z^{-1} = \frac{1}{z} = \frac{1}{5+i} = \frac{1}{5+i} X \frac{5-i}{5-i} = \frac{5-i}{5^2-i^2} = \frac{5-i}{26}$

Do it yourself:

1) Write $\frac{3+4i}{5-12i}$ in the x + iy form, hence find its real and imaginary parts:

2) If $z_1 = 3 - 2i$, $z_2 = 6 + 4i$ find $\frac{z_1}{z_2}$ in the rectangular form

6) Write $\overline{3\iota} + \frac{1}{2-i}$ in rectangular form:

$\overline{3\iota} + \frac{1}{2-i} = -3i + \frac{1\,x(2+i)}{(2-i)(2+i)} = -3i + \frac{2+i}{2^2-i^2}$

$-3i + \frac{2+i}{5} = \frac{15i+2+i}{5} = \frac{2-14i}{5}$

7) If z = x + iy, write $Re(i\bar{z})$ in rectangular form

$Re(i\bar{z}) = Re(i(x-iy)) = Re(ix+y) = y$

Do it yourself:

1) (5 + 9i) + (2 -4i) 2) $Re(1/z)$

8) Find $\left|\frac{i\,(2+i)^3}{(1+i)^2}\right|$

$\left|\frac{i\,(2+i)^3}{(1+i)^2}\right| = \frac{|i||2+i|^3}{|1+i|^2} = \frac{1\,(\sqrt{4+1})^3}{(\sqrt{1+1})^2} = \frac{\sqrt{5}^3}{\sqrt{2}^2} = \frac{5\sqrt{5}}{2}$

Do it yourself:

Find the modulus of the following:

1) $\frac{2\,i}{3+4i}$ 2) $(1-i)^{10}$

9) If $|z| = 3$, prove that $7 \leq |z+6-8i| \leq 13$

$*\ ||z_1| - |z_2|| \leq \ \leq |z_1| + |z_2|$

Let $z_1 = z, z_2 = 6 - 8i$ $|z_2| = \sqrt{36+64} = 10$

$|z| - |6-8i| \leq |z+6-8i| \leq |z| + 10$

$|3-10| \leq |z+6-8i| \leq 3 + 10$

$7 \leq |z+6-8i| \leq 13$

Do it yourself:

If $|z| = 2, P.T. 3 \leq |z+3+4i| \leq 7$

10) Find the square root of the following:

i) -6 + 8i ii) -5 – 12i

$\sqrt{a+ib} = \pm\left[\sqrt{\frac{/z/+a}{2}}\right] + \frac{ib}{|b|}\sqrt{\frac{/z/-a}{2}}$

Let z = -6+8i a = -6, b = 8 $|z| = \sqrt{36+64}$ = 10

$\sqrt{-6+8i} = \pm\left[\sqrt{\frac{10+(-6)}{2}} + \frac{i8}{|8|}\sqrt{\frac{10-(-6)}{2}}\right] = [\sqrt{2} + i\sqrt{8}]$

ii) z = -5-12i

a = -5, b = -12 $|z|\sqrt{25+144}$ = 13

$\sqrt{-5+12i} = \pm\left[\sqrt{\frac{13+(-5)}{2}} + \frac{i\,(-12)}{|-12|}\sqrt{\frac{13-(-5)}{2}}\right] = \pm[2-3i]$

Do it yourself:

$\sqrt{6+8i}$, $\sqrt{4+3i}$ - find

11. Obtain the cartesian form of the locus of z = x + iy in each of the following:

i) $[Re\ (iz)]^2 = 3$ ii) $\bar{z} = \bar{z}^1$

z = x + iy

(z = i(x + iy) = (ix – y) = -y + ix

$[Re\ (iz)]^2 = (-y)^2 = 3$

$\therefore y^2 - 3 = 0$

ii) $\bar{z} = \bar{z}^1$

x – iy = $\frac{1}{x+iy}$

(x + iy) (x – iy) = i

$x^2 + y^2 = 1$

Do it yourself:

i) $|z| = |z - i|$ ii) $|z + i| = |z - 1|$

12. Show that the following equations represent a circle and find its centre and radius

1. $|2z + 2 - 4i| = 2$

   (÷2) $|z + 1 - 2i| = 1$

   $|z - (-1 + 2i)| = 1$ It is of the form $|z - z_0| = r$ and so it represents a circle

   Centre = -1+2i = (-1, 2) radius = 1

2. $|3z - 6 + 12i| = 8$

   (÷3) $|z - 2 + 4i| = 8/3$

   $|z - (2 - 4i| = 8/3$

   Centre= 2 – 4i = (2, -4)radius $= 8/3$

Do it yourself:

1. $|z - 2 - i| = 3$
2. $|3z - 5 + i| = 4$

13. If $z = 3 + 2i$ then show that $z,\ iz$ and $z + iz$ form the vertices of an isosceles right triangle

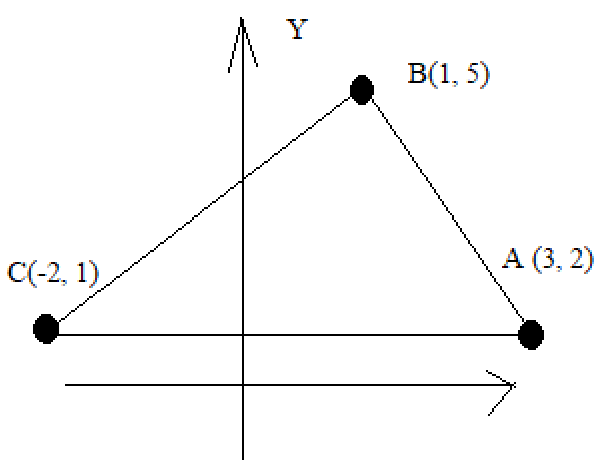
$AB^2 = |(x + iz) - 2|^2 = |-2 + 3i|^2 = \sqrt{4+9}^2 = \sqrt{13}^2 = 13$

$BC^2 = |iz - (z + iz)|^2 = |-3 + 2i|^2 = 13$

$CA^2 = |z - iz|^2 = |5 - i|^2 = 26$

$AB^2 + BC^2 = CA^2$ and AB = BC

∴ ΔABC is an isosceles right triangle.



THREE MARKS QUESTION:

1. Triangle inequality

For any two complex numbers z_1, z_2

$|z_1 + z_2| \leq |z_1| + |z_2|$

Proof: $|z_1 + z_2|^2 = (z_1 + z_2)(\overline{z_1 + z_2})$

$= (z_1 + z_2)(\overline{z_1} + \overline{z_2})$

$= z_1\overline{z_1} + z_2\overline{z_2} + z_1\overline{z_2} + z_2\overline{z_2}$

$= |z_1|^2 + |z_2|^2 + z_1\overline{z_2} + \overline{z_2\overline{z_2}}$

$= |z_1|^2 + |z_2|^2 + 2Re(z_1\overline{z_2})$

$\leq |z_1|^2 + |z_2|^2 + 2\,(z_1\overline{z_2})$

$\leq |z_1|^2 + |z_2|^2 + 2\,|z_1||\overline{z_2}|$

$\leq [|z_1| + |z_2|]^2$

Taking sq.root, $|z_1 + z_2| \leq |z_1| + |z_2|$

2. Find the values of the real numbers x and y, if the complex numbers (3-i)x - 2(2-i)y + 2i + 5 and 2x + (-1 + 2i) y + 3 + 2i Equating real and imaginary parts

(3-i)x – (2-i)y + 2i+5 = 2x + (-1 + 2i)y + 3 + 2i

3x –ix – 2y + iy + 2i + 5 = 2x – y +2iy + 3 + 2i

3x – 2y + 5 = 2x = y + 3 -x + y + 2 = 2y + 2

x – y + 2 = 0 …(1) -x – y = 0 ………… (2)

(1) + (2) ⇒ -2y + 2 = 0

y = 1 from equ. (2)–x -1 = 0

x = -1

Do it yourself:

Find the value of the real numbers and y if the complex number (2+i)x + (1-i)y + 2i -3and x+(-1+2i)y + 1 + i are equal

3. Find the additive and multiplicative inverse of -3-4ilet z = -3 -4i

Additive inverse of z => -z = 3 + 4i

Multiplicative inverse of z => $\frac{1}{z} = \frac{1}{-3-4i}$

$= \frac{1 \times (-3+4i)}{(-3-4i)(-3+4i)} = \frac{-3+4i}{9+16} = \frac{-3+4i}{25}$

Do it yourself:

1). Find the additive and multiplicative inverse of 2+5i

2) If z_1=3, z_2= -7i , z_3=5+4i then prove that $z_1(z_2+z_3) = z_1 z_2 + z_1 z_3$

4. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$ If v = 3 – 4i and w = 4+3i find u – in rectangular form.

$$\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$$

$$= \frac{1}{3-4i} + \frac{1}{4+4i}$$

$$\frac{1}{u} = \frac{3+4i}{9+16} + \frac{4-3i}{16+9} = \frac{7+i}{25}$$

$$\therefore u = \frac{25}{7+i} = \frac{75(7-i)}{(7+i)(7-i)} = \frac{25(7-i)}{50} = \frac{7-i}{2}$$

5. Which one of the points 10-8i, 11+6i is closest to 1 + i

Let z = 1+i, z_1 = 10-8i, z_2 = 11 + 6i

$$|z - z_1| = |(1+i)-(10-8i)| = |-9+9i| = \sqrt{81+81} = \sqrt{162}$$

$$|z - z_2| = |(1+i)-(11-6i)| = |-10-5i| = \sqrt{100+25} = \sqrt{125}$$

$$\sqrt{125} < \sqrt{162}$$

z_2 = 11 + 6i is closest to 1 +i

6. If z_1, z_2 and $z_3$are complex numbers such that $|z_1| = |z_2| = |z_3|$ $=|z_1 + z_2 + z_3| = 1$find the value of$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$

$$|z_1| = |z_2| = |z_3| = 1$$

$$|z_1|^2 = 1 \quad \Rightarrow z_1\overline{z_1} = 1$$

$$\overline{z_1} = 1/z_1, \Rightarrow \overline{z_2} = 1/z_2$$

$$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = |\overline{z_1} + \overline{z_2} + \overline{z_3}| = |\overline{z_1 + z_2 + z_3}| = |z_1 + z_2 + z_3| = 1$$

7. Show that the equation$z^2 = \bar{z}$has four solutions:.

$z^2 = \bar{z}$

$|z^2| = |\bar{z}| = |z|$

$|z^2| = |z|$ $\quad\quad |z| = 1$

$|z|(|z|-1) = 0$ $\quad\quad |z^2| = 1$

$|z| = 0$ $\quad\quad z\bar{z} = 1$

$z = 0$ is a solution $\quad\quad$ ∴ $\bar{z} = 1/z$

$z^2 = \bar{z} = \frac{1}{z}$

$z^3 = 1$ $\quad z^3 - 1 = 0$It has 3 non zero solution including zero solution, there are four solutions.

Do it yourself:

1. Show that the equation $z^3 + 2\bar{z} = 0$ has five solution

Five mark Questions

1. Prove the following:

1. $(2+i\sqrt{3})^{10}-(2-i\sqrt{3})^{10}$ is purely imaginary

$z = \left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$

$\bar{z} = \overline{(2+\imath\sqrt{3})^{10}-(2-\imath\sqrt{3})^{10}}$

$= \overline{(2+\imath\sqrt{3})^{10}} - \overline{(2-\imath\sqrt{3})^{10}}$

$= \overline{(2+\imath\sqrt{3})}^{10}-\overline{(2-\imath\sqrt{3})}^{10}$

$= (2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$

$= -\left[(2+i\sqrt{3})^{10}-(2-i\sqrt{3})^{10}\right] = -z$

$\therefore\ \bar{z} = -z$ z is purely imaginary

2. $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real

$\frac{19-7i}{9+i} = \frac{19-7i}{9+i} \times \frac{9-i}{9-i} = \frac{171-19i-63i+7i^2}{81+1}$

$= \frac{164-82i}{82} = \frac{82(2-i)}{82} = 2-i$

$\frac{20-5i}{7-6i} = \frac{20-5i}{7-6i} \times \frac{7+6i}{7+6i} = \frac{140-120i-35i+30i^2}{49+36}$

$= \frac{170+85i}{85} = \frac{85(2+i)}{85} = 2+i$

$\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12} = (2-i)^{12} + (2+i)^{12} = z$ say

$\bar{z} = \overline{(2-\imath)^{12} + (2+\imath)^{12}} = \overline{(2-\imath)^{12}} + \overline{(2-\imath)^{12}}$

$= \overline{(2-\imath)}^{12} + \overline{(2+\imath)}^{12}$

$=(2+i)^{12} + (2-i)^{12} = z$

$\bar{z} = z \Rightarrow$ z is purely real

Do it yourself:

Prove 1. $(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is purely real and

2. $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8-i}{1+2i}\right)^{15}$ is purely imaginary

2. Show that the points 1, $\frac{-1}{2}+\frac{i\sqrt{3}}{2}$ and $\frac{-1}{2}-\frac{i\sqrt{3}}{2}$ are the vertices of an equilateral triangle

$z_1 = 1$, $z_2 = \frac{-1+i\sqrt{3}}{2}$, $z_3 = \frac{-1-i\sqrt{3}}{2}$

$|z_1 - z_2| = \left|1-\left(\frac{-1+i\sqrt{3}}{2}\right)\right| = \left|\frac{3-i\sqrt{3}}{2}\right| = \frac{\sqrt{9+3}}{2} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$|z_2 - z_3| = \left|\left(\frac{-1+i\sqrt{3}}{2}\right)-\left(\frac{-1-i\sqrt{3}}{2}\right)\right| = \left|\frac{0+2i\sqrt{3}}{2}\right| = \sqrt{0^2+\sqrt{3}^2} = = \sqrt{3}$

$|z_3 - z_1| = \left|\frac{1}{2}-\frac{i\sqrt{3}}{2}-1\right| = \left|\frac{-3-i\sqrt{3}}{2}\right| = \frac{\sqrt{9+3}}{2} = \sqrt{3}$

Since the sides are equal, the given points form an equilateral triangle.

3. Let z_1 , z_2, z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = r > 0$ and $z_1 + z_2 + z_3 \neq 0$ then prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right| = r$

solution:

$|z_1| = |z_2| = |z_3| = r$

$z_1\overline{z_1} = r^2$

$z_1 = \frac{r^2}{\overline{z_1}}, z_2 = \frac{r^2}{\overline{z_2}}, z_3 = \frac{r^2}{\overline{z_3}}$

$z_1 + z_2 + z_3 = \frac{r^2}{\overline{z_1}} + \frac{r^2}{\overline{z_2}} + \frac{r^2}{\overline{z_3}} = r^2\left[\frac{\overline{z_2z_3}+\overline{z_1z_3}+\overline{z_1z_2}}{\overline{z_1z_2z_3}}\right]$

$|z_1 + z_2 + z_3| = |r^2|\left[\frac{\overline{z_2z_3}+\overline{z_1z_3}+\overline{z_1z_2}}{\overline{z_1z_2z_3}}\right] = \frac{r^2|z_2z_3+z_1z_3+z_1z_2|}{|z_1||z_2||z_3|}$

$= \frac{r^2[|z_2z_3+z_1z_3+z_1z_2|]}{r^3}$

$r = \frac{|z_2z_3+z_1z_3+z_1z_2|}{|z_1+z_2+z_3|} = \left|\frac{z_2z_3+z_1z_3+z_1z_2}{z_1+z_2+z_3}\right|$

Do it yourself:

If z_1 , z_2 and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$ show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$

4) If z = x + iy is a complex number such that Im$\left[\frac{2z+1}{iz+1}\right] = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+(2iy)}{ix-y+1}$

$= \frac{(2x+1)+(2iy)}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$

Im$\left[\frac{2z+1}{iz+1}\right] = \frac{-x(2x+1)+2y(1-y)}{(1-y)^2+x^2} = 0$

$-2x^2 - x + 2y - 2y^2 = 0$

(∴) $2x^2 + 2y^2 + x - 2y = 0$

5. Obtain the Cartesian form of the locus of z = x + iy in $|z+i| = |z-1|$

$|x+iy+i| = |x+iy-1|$

$|x+i((y+1)| = |(x-1)+iy)|$

$\sqrt{x^2+(y+1)^2} = \sqrt{(x-1)^2+y^2}$

Squaring on both sides $x^2+(y+1)^2 = (x-1)^2+y^2$

$x^2 + y^2 + 2y + 1 = x^2 - 2x + 1 + y^2$

2x + 2y = 0

(÷ 2) x + y = 0

Do it yourself:

If z = x + iy is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus of ,zis real axis

Chapter – 3 (Theory of Equation)

1. Quadratic equation is $ax^2 + bx + c = 0$

The roots are α,β

$\Sigma_1 = \alpha + \beta = -\frac{b}{a}$

$\Sigma_2 = \alpha\beta = \frac{c}{a}$

If roots are given then the equation is

$x^2 - \Sigma_1 x + \Sigma_2 = 0$

2. Cubic equation is $ax^3 + bx^2 + cx + d = 0$

The roots are α,β,γ

$\Sigma_1 = \alpha + \beta + \gamma = -\frac{b}{a}$

$\Sigma_2 = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\Sigma_3 = \alpha\beta\gamma = -\frac{d}{a}$

If roots are given then the equation is

$x^3 - \Sigma_1 x^2 + \Sigma_2 x - \Sigma_3 = 0$

3. Fourth degree equation is

$ax^4 + bx^3 + cx^2 + dx + e = 0$

The roots are α,β,γ,δ

$\Sigma_1 = \alpha + \beta + \gamma + \delta = -\frac{b}{a}$

$\Sigma_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$

$\Sigma_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$

$\Sigma_4 = \alpha\beta\gamma\delta = \frac{e}{a}$

If roots are given then the equation is

$$x^4 - \Sigma_1 x^3 + \Sigma_2 x^2 - \Sigma_3 x + \Sigma_4 = 0$$

1. If α,β are the roots of the equation $17x^2 + 43x - 73 = 0$ construct a quadratic equation whose roots are $\alpha + 2$, $\beta + 2$

Solution:

$17x^2 + 43x - 73 = 0$

a = 17, b = 43, c = -73

$\Sigma_1 = \alpha + \beta = -\frac{b}{a} = -\frac{43}{17}$

$\Sigma_2 = \alpha\beta = \frac{c}{a} = -\frac{73}{17}$

Given roots are $\alpha + 2$, $\beta + 2$

$\Sigma_1 = \alpha + 2 + \beta + 2 = \alpha + \beta + 4 = -\frac{43}{17} + 4 = -\frac{43+68}{17}$

$= \frac{25}{17}$

$\Sigma_2 = (\alpha + 2)(\beta + 2) = \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha + \beta) + 4$

$= -\frac{73}{17} + 2\left(\frac{-43}{17}\right) + 4$

$= \frac{-73-86+68}{17} = \frac{-91}{17}$

∴ equation $x^2 - \Sigma_1 x + \Sigma_2 = 0$

$x^2 - \frac{25}{17} \times \frac{-91}{17} = 0$

x17 $\quad 17x^2 - 25x - 91 = 0$

2. If α, β are roots of $2x^2 - 7x + 13 = 0$ construct a quadratic equation whose roots are α^2, β^2 (Eg: 3.2)

Solution:

$2x^2 - 7x + 13 = 0$

a = 2, b = -7, c = 13

$\Sigma_1 = \alpha + \beta = \frac{-b}{a} = -\left(\frac{-7}{2}\right) = \frac{7}{2}$

$\Sigma_2 = \alpha\beta = \frac{c}{a} = \frac{13}{2}$

To form the equation whose roots are α^2, β^2

$$\Sigma_1 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{7}{2}\right)^2 - 2\left(\frac{13}{2}\right) = \frac{49}{4} - 13$$

$$= \frac{49 - 52}{4} = \frac{-3}{4}$$

$$\Sigma_2 = \alpha^2\beta^2 = (\alpha\beta)^2 = \left(\frac{13}{2}\right)^2 = \frac{169}{4}$$

∴ equation $x^2 - \Sigma_1 x + \Sigma_2 = 0$

$$x^2 - \left(\frac{-3}{4}\right)x + \frac{169}{4} = 0$$

$$x^2 - \frac{3}{4}x + \frac{169}{4} = 0$$

x 4 $\quad 4x^2 + 3x + 169 = 0$

3. Find the polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root

(Eg 3.2 (4))

Solution: $\sqrt{5} - \sqrt{3}$ is a root

$\sqrt{5} + \sqrt{3}, -\sqrt{5} + \sqrt{3}, -\sqrt{5} - \sqrt{3}$ are also roots

$x = \sqrt{5} - \sqrt{3}$

squaring both sides: $[(a-b)^2 = a^2 + b^2 - 2ab]$

$$x^2 = \left(\sqrt{5} - \sqrt{3}\right)^2 = \left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 - 2\sqrt{5}\sqrt{3}$$

$$x^2 = 5 + 3 - 2\sqrt{5}\sqrt{3}$$

$$x^2 = 8 - 2\sqrt{5}\sqrt{3}$$

$$x^2 - 8 = -2\sqrt{5}\sqrt{3}$$

Again squaring both sides

$$(x^2 - 8)^2 = \left(-2\sqrt{5}\sqrt{3}\right)^2$$

$$x^4 + 64 - 2x^2(8) = 4(5)(3)$$

$$x^4 + 64 - 16x^2 = 60$$

$$\therefore x^4 - 16x^2 + 64 - 60 = 0$$

$$x^4 - 16x^2 + 4 = 0$$

4. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root

(Eg: 3.10)

$x = \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$

Squaring both sides $x^2 = \frac{\sqrt{2}}{\sqrt{3}}$

Again squaring both sides

$$x^4 = \frac{2}{3}$$

$$3x^4 - 2 = 0$$

5. Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x

(Eg 3.11)

Solution::

Discriminant $\Delta = b^2 - 4ac$

Here a = 2, b = -6, c = 7

$\Delta = (-6)^2 - 4(2)(7) = 36 - 56$

$\therefore \Delta = -20 < 0$

$\because \Delta < 0$ the roots are imaginary

6. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k

(Eg 3.12)

Solution: Equal roots $\Delta = b^2 - 4ac$

Here a = 1 b = 2(K+2), c = 9k

Equal roots $\Delta = 0$

$\therefore [2(k+2)]^2 - 4(1)(9k) = 0$

$4(k+2)^2 - 4(9k) = 0$

÷4 $(k+2)^2 - 9k = 0$

$K^2 + 4 + 4k - 9k = 0$

$K^2 - 5k + 4 = 0$

$(k-4)(k-1) = 0$

$k = 4, \; k = 1$

4

-4 -1 | -5

7. Solve the cubic equation $2x^3 - 9x^2 + 10x = 3$ (Eq.3.3 – 6(i))

Solution:

$2x^3 - 9x^2 + 10x = 3$First rewrite the given equation as below

$2x^3 - 9x^2 + 10x - 3 = 0$

The Co-efficient are 2 ,– 9, 10, -3

Sum of the coefficients = 2 – 9 +10 – 3 = 12-12 = 0

$\therefore x = 1$then is root

If sum of the coefficients

x = 1	2	-9	10	-3
	0	2	-7	3
	2	-7	3	0

	6	
-6	-1	-7
2	2	
-3	-1/3	

$2x^2 - 7x + 3 = 0$

$(x - 3)(x - ½) = 0 \quad \therefore x = 3, \ x = ½$

Solution: x = 1, x = 3, x =1/2

8. Solve the equation:$x^3 - 3x^2 - 33x + 35 = 0$ (Eg3.17)

Solution:

The co-efficients 1 -3 -33 35

sum of the roots 1-13-33+35 = 36 – 36 = 0

∴ x = 1 is a real

x = 1	1	-3	-33	35
	0	1	-2	-35
	1	-2	-35	0

$x^2 - 2x - 35 = 0$

	-35	
-7	5	-2

$(x - 7)(x + 5) = 0$

x = 7 , x = 5

∴ Solution: x = 1, x = 7, x = -5

9. Solve the cubic equation $8x^3 - 2x^2 - 7x + 3 = 0$ (Eg3.3 6(ii))

Solution:

The co-efficient are 8 -2 -7 3

S₁ S₂

$S_1 = 8 - 7 = 1$, $S_2 = -2 + 3 = 1$

$S_1 = S_2$ ∴ $x = -1$ is a root

x = -1				
	8	-2	-7	3
	0	-8	10	-3
	8	-10	3	0

	24	
-6	-4	-10
-6/8	-4 / 8	
-3/4	-1/2	

$8x^2 - 10x + 3 = 0$

$\left(x - \frac{3}{4}\right)\left(x - \frac{1}{2}\right) = 0$

$x = \frac{3}{4}$, $x = \frac{1}{2}$

Solution: $x = -1$, $x = \frac{3}{4}$, $x = \frac{1}{2}$

10. Solve the cubic equation $2x^3 + 11x^2 - 9x - 18 = 0$ (Eg: 3.18)

The co-efficient are 2 11 -9 -18

S₁ S₂

$S_1 = 2 - 9 = -7$, $S_2 = 11 - 18 = -7$

$S_1 = S_2$ ∴ $x = -1$ is a root

x = -1				
	2	11	-9	-18
	0	-2	-9	18
	2	9	-18	0

	36	
12	-3	9
12	-3	
2	2	
6	-3	
	2	

$2x^2 + 9x - 18 = 0$

$(x + 6)\left(x - \frac{3}{2}\right) = 0$

$x = -6$, $x = \frac{3}{2}$

∴ Solution: $x = -6$, $x = \frac{3}{2}$, $x = -1$

11. Solve the equation: $7x^3 - 43x^2 = 43 - 7$ (Eg: 3.27)

Solution: $7x^3 - 43x^2 = 43 - 7$

Rewriting the equation in correct order $7x^3 - 43x^2 - 43x + 7 = 0$

The coefficients are 7 -43 -43 7

S1 S2

$S_1 = 7 - 43 = -36$, $S_2 = -43 + 7 = -36$

$S_1 = S_2$ ஃ $x = -1$ is a root

x = -1				
	7	-43	-43	7
	0	-7	50	-7
	7	-50	7	0

	49	
-49	-1	-50
-49/7	-1/7	
-7	-1/7	

$7x^2 - 50x + 7 = 0$

$(x - 7)\left(x - \frac{1}{7}\right) = 0$

$x = 7, \quad x = \frac{1}{7}$

ஃ Solution: $x = -1$, $x = 7$, $x = \frac{1}{7}$

12. Solve : $x^4 + 3x^3 - 3x - 1 = 0$ (Eg3.5 5(ii))

Solution:

The coefficients are 1 3 0 -3 -1 [Note

Sum of the coefficients $1 + 3 + 0 - 3 - 1 = 4 - 4 = 0$

x^2 term is missing]

ஃ $x = 1$ is a root

S2

1 3 0 -3 -1

S1

$S_1 = 1 + 0 - 1 = 0$, $S_2 = 3 - 3 = 0$

$S_1 = S_2$ ஃ $x = -1$ is a root

x = 1	1	3	0	-3	-1
	0	1	4	4	1
x = -1	1	4	4	1	0
	o	-1	-3	-1	
	1	3	1	0	

$x^2 + 3x + 1 = 0$

$a = 1,\ b = 3,\ c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{9-4}}{2} \quad = \frac{-3 \pm \sqrt{5}}{2}$$

∴ Solution: $x = 1,\ x = -1, x = \frac{-3+\sqrt{5}}{2} \quad , x = \frac{-3-\sqrt{5}}{2}$

13. Solve the cubic equation $2x^3 - x^2 - 18x + 9 = 0$ if sum of its two roots vanishes:

(Eg3.3(1))

Solution clearly sum $2 - 1 - 18 + 9 \neq 0$

$x = 1$ is not a root

$S_1 \neq S_2$ ∴ $x = -1$ is not a root

$x = 2,\ x = -2$ are also not roots

By trail put $x = 3$

x = 3				
	2	-1	-18	9
		6	15	-9
	2	5	-3	0

Reaminder is 0 $x = 3$is a root

Put$x = -3$

x= -3			
	2	5	-3
		-6	3
	2	-1	0

Remainder is 0 $x = -3$is also a root

Note Clearly sum of two of its roots $3 - 3 = 0$ vanishes

$2x - 1 = 0$

$2x = 1$

$x = ½$

∴ solution $x = 3,\ x = -3,\ x = ½$

14. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$ if it is known that $1+2i$ and $\sqrt{3}$ are two of its roots

(Eg 3.3 (5))

Solution :

The root are $1+2i,\ 1-2i,\ \sqrt{3},\ -\sqrt{3},\ \alpha,\ \beta$

$$\Sigma_1 = 1 + 2i + 1 - 2i + \sqrt{3} - \sqrt{3} + \alpha + \beta = \frac{constant}{Coefficient of x^6}-$$

$$2 + \alpha + \beta = \frac{-(-3)}{1} = 3$$

$\alpha + \beta = 1$ …………… (1)

$$\Sigma_6 = (1+2i)(1-2i)(\sqrt{3})(-\sqrt{3}) + \alpha\beta = \frac{coefficient of x^5}{Coefficient of x^6}-$$

$(12 + 22)(-3)\alpha\beta = 135 / 1 = 135$

$(1 + 4)(-3)\alpha\beta = 135$

$5(-3)\alpha\beta = 135$

$\alpha\beta = \frac{135}{-15} = -9$

$\alpha\beta = -9$

$x^2 - (\alpha\beta)x + \alpha\beta = 0$

$x^2 - x - 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{(-1)^2-4(1)(-9)}}{2} = \frac{1 \pm \sqrt{1+36}}{2} = \frac{1 \pm \sqrt{37}}{2}$$

Solution : $1+2i,\ 1-2i, \sqrt{3}, -\sqrt{3}, \frac{1+\sqrt{37}}{2}, \frac{1-\sqrt{37}}{2}$

15. Solve the equation: $x^4 - 9x^2 + 20 = 0$ (Eg: 3.16)

Put $x^2 = y$

$y^2 - 9y + 20 = 0$

$(y-5)(y-4) = 0$

$y = 5,\ y = 4$

	20	
-5	-4	-9

$y = 5$ If $x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

$y = 4$ If $x^2 = 4 \Rightarrow x = \pm 2$

Solution $x = 2, -2, \sqrt{5}, -\sqrt{5}$

Solve the equation: $y^4 - 14x^2 + 45 = 0$ Eg: 3.3(7)

	45	
-9	-5	-14

Put $x^2 = y$

$y^2 - 14y + 45 = 0$

$(y - 9)(y - 5) = 0$

$y = 9, \; y = 5$

$y = 9$ If $x^2 = 9 \Rightarrow x = \pm 3$

$y = 5$ If $x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

Solution $x = 3, -3, \sqrt{5}, -\sqrt{5}$

16. Solve: $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ (Eg3.5.5 (i)

Solution: This is a reciprocal equation

x = 2put

x = 2	6	-35	62	-35	6
		12	-46	32	-6
	6	-23	16	-3	0

Remainder is 0 x = 2is a root

x = 3is a root

Since the given equation is a reciprocal equation

x = 2, x = 3 are roots x = ½, x =1/3are also root

ஃ solution x = 2, x =3, x = ½, x = 1/3

17. Solve $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$, x = 1/3if it is known that is a solution

Solution: This is a reciprocal equation

Given x = 1/3 is a root x = 3is a root

x = 3	6	-5	-38	-5	6	
	0	18	39	3	-6	
x = -2	6	13	1	-2	0	x = -2
	o		-12	-2		2
	6	1	-1	0		

Remainder0 x = -2is a solution

The given equation is a reciprocal equation x = -1/2is also a solution

ஃSolution x = 3, x =1/3, x = -2, x = ½

18. Solve : $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ (Eg: 3.28)

Solution: For this problem, let us remainder few steps .

Put $x + \frac{1}{x} = y$

$y^2 - 10y + 24 = 0$

	24		
	-4	-6	-10

$(y - 4)(y - 6) = 0$

y=4 ,y=6

(i) If y = 4 $x + \frac{1}{x} = 4 \Rightarrow \frac{x^2+1}{x} = 4 \Rightarrow x^2 + 1 = 4x$

$x^2 - 4x + 1 = 0$ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ a = 1, b = -4, c = 1

$x = \frac{4 \pm \sqrt{(-4)^2-4(1)(1)}}{2} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm \sqrt{4 \times 3}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \frac{2(2 \pm \sqrt{3})}{2} = (2 \pm \sqrt{3})$

(ii) If y = 6 $\Rightarrow x + \frac{1}{x} = 6 \Rightarrow \frac{x^2+1}{x} = 6 \Rightarrow x^2 + 1 = 6x$

$x^2 - 6x + 1 = 0$ $x = \frac{6 \pm \sqrt{(-6)^2-4(1)(1)}}{2}$

$x = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm \sqrt{16 \times 2}}{2}$

$= \frac{6 \pm 4\sqrt{2}}{2} = \frac{2(3 \pm \sqrt{2})}{2} = (3 \pm \sqrt{2})$

Solution: $2 + \sqrt{3},\ 2 - \sqrt{3}\ ,\ 3 + \pm\sqrt{2}\ ,\ 3 - \sqrt{2}$

19. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$ (Ex: 3.6(1))

Solution : $P(x) = 9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

P(x) + - + - + + + + +

1 2 3 4

Number of sign changes in P(x) is 4

Maximum number of positive roots is 4

P(-x): - - - - - - + - +

1 2 3

Number of sign changes in P(-x) is 3

Maximum number of negative roots is 3

20. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 =$ (Eg: 3.6(4))

Solution : $P(x) = 9x^9 - 4x^8 + 4x^7$

P(x) = + - -

1

Number of sign changes in P(x) is 1

Maximum number of positive roots is 1

P(-x) = - - +

1

Number of sign changes in P(-x) is 1

Maximum number of negative roots is 1

21. Find the exact number of real roots and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$ (Eg: 3.6(5))

Solution : P(x) = + + + + +

No sign change

No positive real root

P(-x) = - - - - -

No sign change

No negative real root

Since there is no constant term x = 0 is a root

∴ There are 8 imaginary rotos

(degree is 9 – one zero root = 8)

22. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2 = 0$has atleast six imaginary roots. (Eg: 3.30)

Solution: P(x) = + + - - +

1 2

Number of sign changes in P(x) is 2

Maximum number of positive roots is 2

P(-x) = - - - - +

1

Number of sign changes in P(-x) is 1

Maximum number of negative roots is 1

There is a constant term (2) in P(x)

Zero is not a root

Minimum number of imaginary roots is

$$9 - (2 + 1) = 9 - 3 = 6$$

4. Inverse Trigometric Functions

Function	Domain	Range
sin^{-1}	$[-1,1]$	$\left[-{}^{\pi}/_{2}\ ,\ {}^{\pi}/_{2}\right]$
cos^{-1}	$[-1,1]$	$[0\ ,\ \pi]$
tan^{-1}	R	$\left(-{}^{\pi}/_{2}\ {}^{\pi}/_{2}\right)$
$cosec^{-1}$	$R\ \backslash\ (-1,1)$	$\left[-{}^{\pi}/_{2}\ {}^{\pi}/_{2}\right]$ / {0}
sec^{-1}	$R\ \backslash\ (-1,1)$	$[0\quad \pi]\ \backslash\ \{{}^{\pi}/_{2}\}$
cot^{-1}	R	$(0\quad \pi)$

If Y = A sinαx then period $=\frac{2\pi}{|\alpha|}$ and amplitude$|A|$

2 & 3 Mark

1. Find the value

$$sin^{-1}\left(\sin {}^{2\pi}/_{3}\right)$$
$$=sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)$$
$$=sin^{-1}\left(\sin\left({}^{\pi}/_{3}\right)\right)$$
$$={}^{\pi}/_{3}\in\left[-{}^{\pi}/_{2},{}^{\pi}/_{2}\right]$$

2. $sin^{-1}\left(\sin\frac{5\pi}{4}\right)$Find the value

$$=sin^{-1}\left(\sin\left(\pi+\frac{\pi}{4}\right)\right)$$
$$=sin^{-1}\left(-\sin\left({}^{\pi}/_{4}\right)\right)$$
$$=sin^{-1}\left(\sin\left(-{}^{\pi}/_{4}\right)\right)$$
$$=\ -{}^{\pi}/_{4}\in\left[-{}^{\pi}/_{2}\ ,{}^{\pi}/_{2}\right]$$

3. Find the value

$$cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$$
$$=cos^{-1}\left(\cos\left(\pi+\frac{\pi}{6}\right)\right)$$
$$=cos^{-1}\left(-\cos\frac{\pi}{6}\right)$$
$$=cos^{-1}\left(\cos\left(\pi-\frac{\pi}{6}\right)\right)$$
$$=cos^{-1}\left(\cos\frac{5\pi}{6}\right)$$
$$=\frac{5\pi}{6}\in\ [0\ ,\pi]$$

4. Find the value: $tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

$$tan^{-1}\left(\tan\frac{5\pi}{4}\right) = tan^{-1}\left(\tan\left(\pi+\frac{\pi}{4}\right)\right)$$

$$= tan^{-1}\left(\tan {}^{\pi}/_{4}\right)$$

$$= {}^{\pi}/_{4} \in \left(-{}^{\pi}/_{2}, {}^{\pi}/_{2}\right)$$

5. Find the principal value

$$sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$sec^{-1}\left({}^{2}/_{\sqrt{3}}\right) = cos^{-1}\left({}^{\sqrt{3}}/_{2}\right) = {}^{\pi}/_{6}$$

$$sec^{-1}x = cot^{-1}\left(\frac{1}{x}\right)$$

6. $cot^{-1}({}^{1}/_{7}) = \theta$ Find the value of $\cos\theta$

$$\theta = cot^{-1}({}^{1}/_{7})$$

$$\cot\theta = {}^{1}/_{7} \Rightarrow \tan\theta = 7$$

$$\sec\theta = \sqrt{1+tan^{2}\theta} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$\cos\theta = \frac{1}{5\sqrt{2}}$$

7. Find the principal value: $cosec^{-1}(-\sqrt{2})$

$$cosec^{-1}(-\sqrt{2}) = sin^{-1}\left({}^{-1}/_{\sqrt{2}}\right)$$

$$= -sin^{-1}\left({}^{1}/_{\sqrt{2}}\right) \qquad sin^{-1}(-x) = -sin^{-1}x$$

$$= {}^{-\pi}/_{4} \qquad x\varepsilon\,[-1, 1]$$

8. $sin^{-1}(2-3x^{2})$in Domain

$$-1 \le 2-3x^{2} \le 1$$

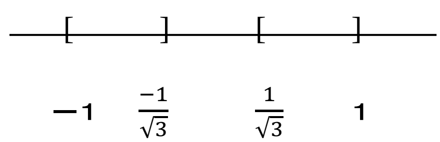
(-2) $$-3 \le -3x^{2} \le -1$$

$(\div 3)$ $$-1 \le -x^{2} \le -{}^{1}/_{3}$$

(-1) $$1 \ge x^{2} \ge {}^{1}/_{3}$$

$$1 \ge |x| \ge {}^{1}/_{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}} \le |x| \le 1$$



$$x\varepsilon\left[-1, -{}^{1}/_{\sqrt{3}}\right] \cup \left[{}^{1}/_{\sqrt{3}}, 1\right]$$

9. $cos^{-1}[\cos(-\pi/6)] \neq -\pi/6$ True? Justify your answer

$cos^{-1}(\cos(-\pi/6)) = cos^{-1}(\cos \pi/6) = \pi/6$

$cos^{-1}(\cos(-\pi/6)) \neq -\pi/6$

10. Find the Domain F(x) $= sin^{-1}\left(\frac{x^2+1}{2x}\right)$

$\left|\frac{x^2+1}{2x}\right| \leq 1$

$x^2 + 1 \leq 2|x|$

$x^2 + 1 - 2|x| \leq 0$

$(|x| - 1)^2 \leq 0$

$|x| - 1 \leq 0$

$x \in [-1, 1]$

11. $cos^{-1}\left(\frac{2+\sin x}{3}\right)$ Find the Domain

$-1 \leq \frac{2+\sin x}{3} \leq 1$

(x3) $-3 \leq 2 + \sin x \leq 3$

(-2) $-5 \leq sin \leq 1$

$-1 \leq \sin x \leq 1$

$-\pi/2 \leq \sin x \leq \pi/2$

$x \in [-\pi/2, \pi/2]$

12. Find the value $sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + cos\frac{5\pi}{9} . sin\frac{\pi}{9}\right)$

$= sin^{-1}\left(\sin\left(\frac{5\pi}{9} + \frac{\pi}{9}\right)\right)$

$= sin^{-1}\left(\sin\left(\frac{6\pi}{9}\right)\right) = sin^{-1}(\sin(2\pi/3))$

$sin^{-1}(\sin(\pi - \pi/3))$

$= \frac{\pi}{3} \in [-\pi/2, \pi/2]$

5 Mark

1. Find the Domain: $F(x) = sin^{-1}\left(\frac{|x|-2}{3}\right) + cos^{-1}\left(\frac{1-|x|}{4}\right)$

$-1 \leq \frac{|x|-2}{3} \leq 1$

$-1 \leq |x| - 2 \leq 3$

$-1 \leq |x| \leq 5$

$|x| \leq 5$

$-5 \leq |x| \leq 5$………. **(1)**

$-1 \leq \frac{1-|x|}{4} \leq 1$

$-4 \leq 1 - |x| \leq 4$

$-5 \leq -|x| \leq 3$

$5 \geq -|x| \geq -3$

$-3 \leq |x| \leq 5$………… **(2)**

2. Find the value: $cos^{-1}\left(cos\left(\frac{4\pi}{3}\right)\right) + cos^{-1}\left(cos\left(\frac{5\pi}{4}\right)\right)$

$= cos^{-1}\left(cos\left(\pi + \frac{\pi}{3}\right)\right) + cos^{-1}\left(cos\left(\pi + \frac{\pi}{4}\right)\right)$

$= cos(\pi + \theta) = cos(\pi - \theta) = -cos\theta$

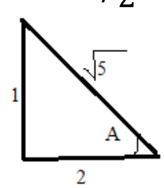
$= cos^{-1}\left(cos\left(\pi - \frac{\pi}{3}\right)\right) + cos^{-1}\left(cos\left(\pi - \frac{\pi}{4}\right)\right)$

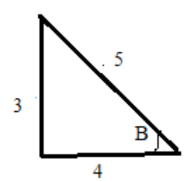
$= cos^{-1}\left(cos\left(\frac{2\pi}{3}\right)\right) + cos^{-1}\left(cos\left(\frac{3\pi}{4}\right)\right)$

$= \frac{2\pi}{3} + \frac{3\pi}{4} = \frac{17\pi}{12}$

3. Find the value $sin\left(tan^{-1}(1/2)\right) - cos^{-1}(4/5)$

$tan^{-1} 1/_2 = A$ $\quad cos^{-1} 4/_5 = B$





$\sin A = 1/_{\sqrt{5}}$

$\cos A = 2/_{\sqrt{5}}$

$\sin B = 3/_5$

$\cos B = 4/_5$

$sin(tan^{-1} 1/2 - cos^{-1} 4/5) = \sin(A - B)$

$= \sin A \cos B - \cos A \sin B$

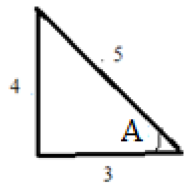
$= \frac{1}{\sqrt{5}} . \frac{4}{5} - \frac{2}{\sqrt{5}} . \frac{3}{5}$

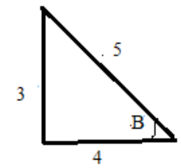
$= \frac{-2}{5\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{-2\sqrt{5}}{25}$

4. Find the value $cos\left(sin^{-1}(4/5)\right) - tan^{-1}(3/4)$

$sin^{-1}\frac{4}{5} = A \quad tan^{-1}\,{}^{3}/_{4} = \quad B$





$\sin A = \,{}^{4}/_{5}$ $\qquad$ $\sin B = \,{}^{3}/_{5}$

$\cos A = \,{}^{3}/_{5}$ $\qquad$ $\cos B = \,{}^{4}/_{5}$

$cos(sin^{-1}4/5 \,-\, tan^{-1}3/4) = \cos(A - B)$

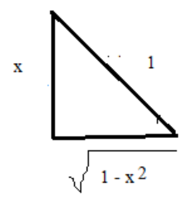
= cos A cos B + sin A sin B

$= \frac{3}{5} \cdot \frac{4}{5} + \frac{4}{5} \cdot \frac{3}{5}$

$= \frac{24}{25}$

5. Verify $\tan(sin^{-1}x) = \frac{x}{\sqrt{1-x^2}}$ -1 < x < 1

LHS: $\tan\left(sin^{-1}\frac{x}{1}\right)$



$= \tan\left(tan^{-1}\frac{x}{\sqrt{1-x^2}}\right)$

$= \frac{x}{\sqrt{1-x^2}}$ $\qquad$ -1 < x < 1

LHS = RHS

6. Find the value: $tan^{-1}(-1) + cos^{-1}(1/2) + sin^{-1}(-1/2)$

$= tan^{-1}(1) + cos^{-1}(1/2) - sin^{-1}(1/2)$

$= \frac{-\pi}{4} + \frac{\pi}{3} - \frac{\pi}{6}$

$= \frac{-\pi}{12}$

7. Find the value $cot^{-1}(1) + sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - sec^{-1}(-\sqrt{2})$

$= tan^{-1}(1) - sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - cos^{-1}\left(-{}^{1}/_{\sqrt{2}}\right)$

$= \frac{\pi}{4} - \frac{\pi}{3} - \left(\pi - {}^{\pi}/_{4}\right)$

$= \frac{\pi}{4} - \frac{\pi}{3} - \pi + {}^{\pi}/_{4}$

$= \frac{\pi}{2} - \frac{\pi}{3} - \pi$

$= \frac{-5\pi}{6}$